How did structure appear in the Universe? A Bayesian approach

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In collaboration with:

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How did structure appear in the Universe?

A joint problem!

- **How did the Universe begin?**
  - What are the statistical properties of the initial conditions?
- **How did the large-scale structure take shape?**
  - What is the physics of dark matter and dark energy?
- Usually these problems are addressed in isolation.
- This talk:
  - A case for physical inference of four-dimensional dynamic states
  - A description of methodology and progress towards enriching the standard for analysis of galaxy surveys
  - From theory to data, from data to theory

FL, Pisani & Wandelt 2014, arXiv:1403.1260

(Lectures Varenna 2013 and Paris École Doctorale for Astronomy and Astrophysics)

Outline

1. Bayesian Inference
2. Chrono-Cosmography
3. The Non-Linear Regime of Structure Formation
4. Cosmic Web Classification
5. The Future

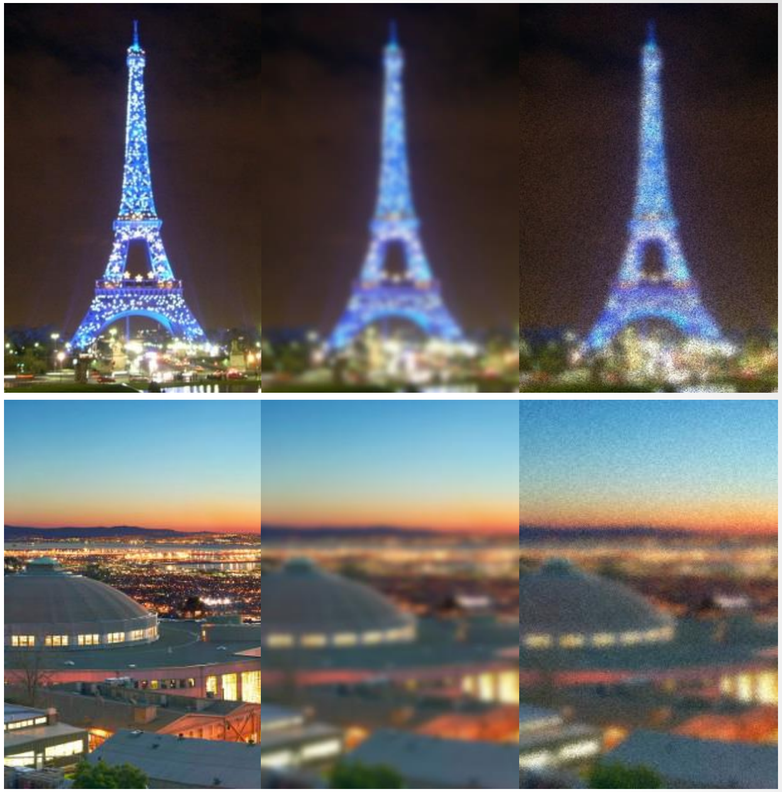
1. Bayesian Inference

- Data assimilation with BORG
- The BORG SDSS run

J. Jasche, B. Wandelt, arXiv:1203.3639.
Bayesian physical reconstruction of initial conditions from large scale structure surveys

J. Jasche, F. Leclercq, B. Wandelt, arXiv:1409.6308.
Past and present cosmic structure in the SDSS DR7 main sample

Why Bayesian inference?



- Why do we need Bayesian inference?

Inference of signals = ill-posed problem

- Incomplete observations: survey geometry, selection effects
- Noise, biases, systematic effects
- Cosmic variance

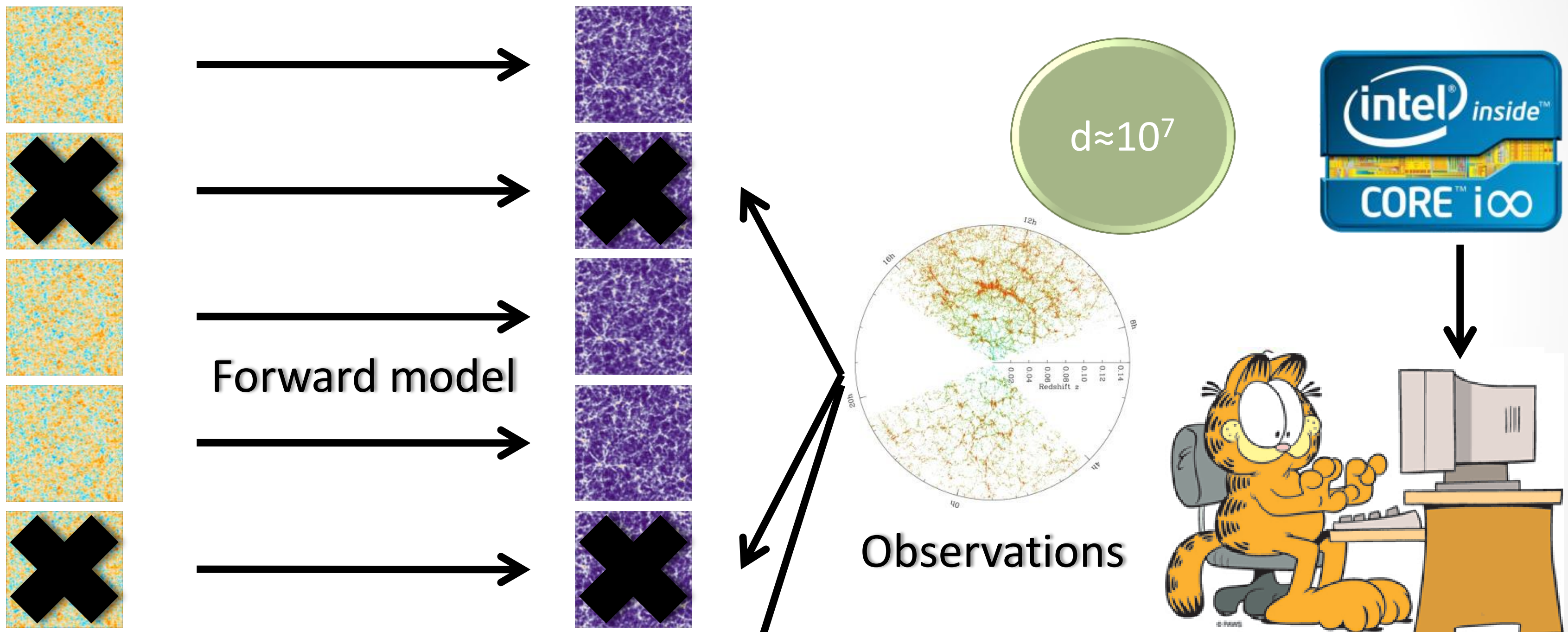
➡ **No unique recovery is possible!**

"What is the formation history of the Universe?" ➡ "What is the probability distribution of possible formation histories (signals) compatible with the observations?"

$$p(s|d)p(d) = p(d|s)p(s)$$

Bayesian forward modeling: the ideal scenario

Forward model = N-body simulation + Halo occupation +
Galaxy formation + Feedback + …



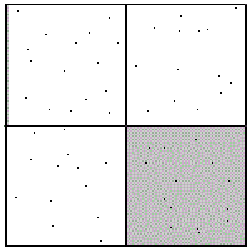
All possible ICs

All possible FCs

We need a very, very, very big computer!

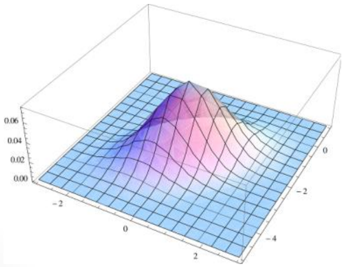
(Parameter) Space: the final frontier

- The "curse of dimensionality" Bellman 1961



dimension	fraction of particles in quadrant of hypercube
1	$2^{-1} = 0.5$
10	$2^{-10} = 9.7 \times 10^{-4}$
100	$2^{-100} = 7.8 \times 10^{-31}$
1000	$2^{-1000} = 9.3 \times 10^{-302}$

  Adding extra dimensions…
  - Exponential increase of the number of particles needed for uniform sampling
  - Exponential increase of sparsity given a fixed amount of particles
- High-dimensional probability distribution functions



Traditional sampling methods will fail

but gradients carry capital information

Hamiltonian Monte Carlo

- Use classical mechanics to solve statistical problems!
  - The potential: $\psi(\mathbf{x}) \equiv -\ln(\mathcal{P}(\mathbf{x}))$
  - The Hamiltonian: $H \equiv \frac{1}{2}\mathbf{p}^T\mathbf{M}^{-1}\mathbf{p} + \psi(\mathbf{x})$

$$(\mathbf{x}, \mathbf{p}) \Rightarrow \left\{ \begin{aligned} \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} &= \frac{\partial H}{\partial \mathbf{p}} = \mathbf{M}^{-1}\mathbf{p} \\ \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} &= -\frac{\partial H}{\partial \mathbf{x}} = -\frac{\partial \psi(\mathbf{x})}{\partial \mathbf{x}} \end{aligned} \right\} \Rightarrow (\mathbf{x}', \mathbf{p}')$$

gradients

$$a(\mathbf{x}', \mathbf{x}) = \mathrm{e}^{-(H'-H)} = 1$$

acceptance ratio unity

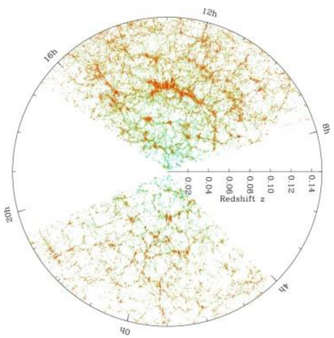
- HMC beats the curse of dimensionality by:
  - Exploiting gradients
  - Using conservation of Hamiltonian

Duane *et al.* 1987

BORG: *Bayesian Origin Reconstruction from Galaxies*

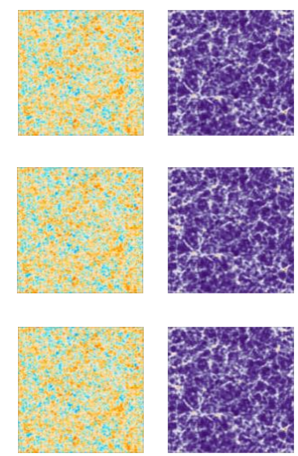
What makes the problem tractable:

- **Sampler**: Hamiltonian Markov Chain Monte Carlo method
- **Physical model**: Second-order Lagrangian perturbation theory (2LPT)



Observations

BORG



Samples of possible 4D states

Jasche & Wandelt 2013, arXiv:1203.3639

see also:
Kitaura 2013, arXiv:1203.4184
Wang, Mo, Yang & van den Bosch 2013, arXiv:1301.1348

2. Chrono-Cosmography

- Past and present cosmic structure in the Sloan volume

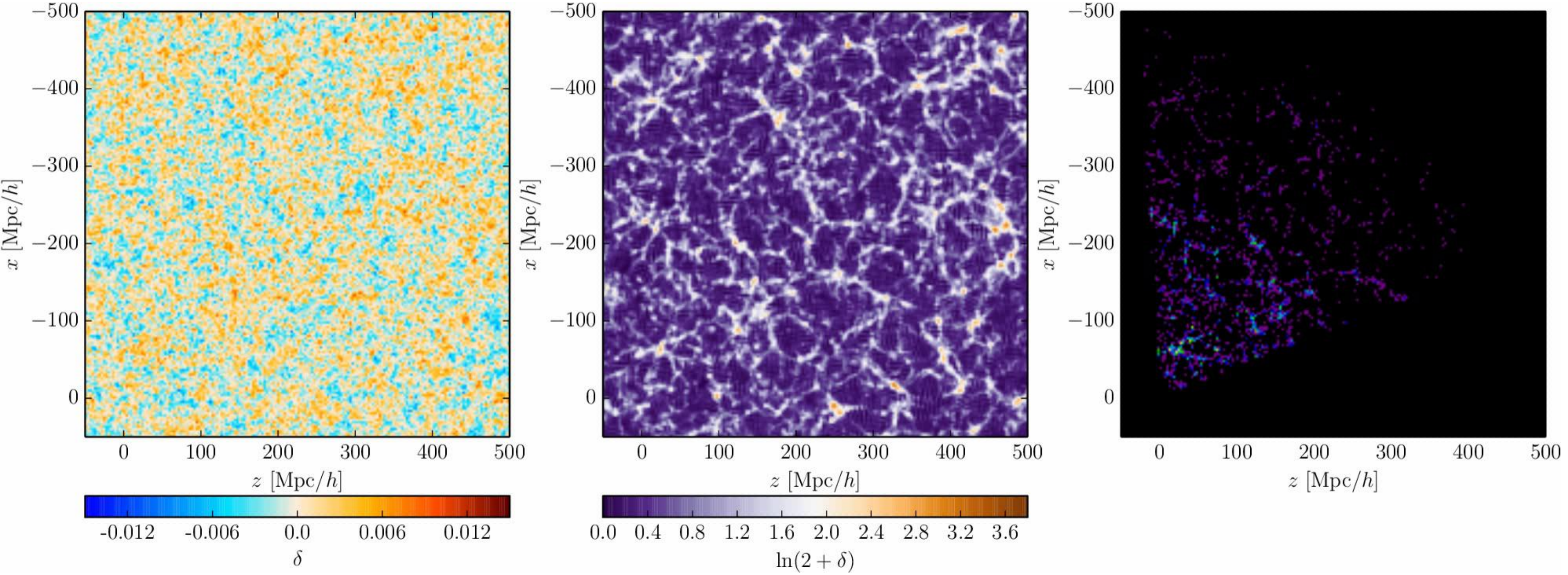
J. Jasche, F. Leclercq, B. Wandelt, arXiv:1409.6308.
Past and present cosmic structure in the SDSS DR7 main sample

The BORG SDSS run

- 463,230 galaxies from the NYU-VAGC based on SDSS DR7
- Comoving cubic box of side length 750 Mpc/*h*, with periodic boundary conditions
- 256^3 grid, resolution 3 Mpc/*h* → ≈ 17 millions parameters
- 12,000 samples, four-dimensional maps
- ≈ 3 TB disk space
- 10 months wallclock time on 16-32 cores

Jasche, FL & Wandelt 2014, arXiv:1409.6308

BORG at work – chronocosmography



Initial conditions | Final conditions | Observations

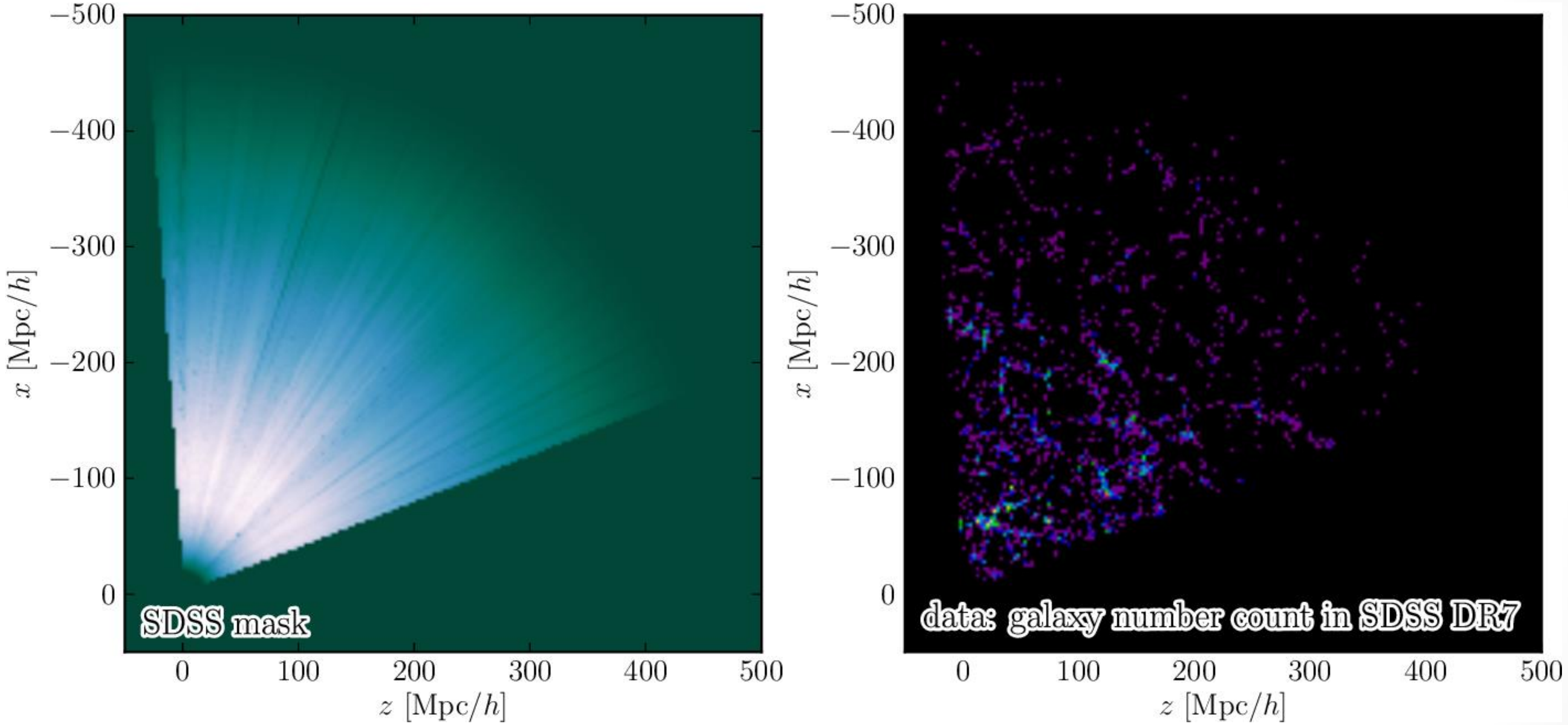
Jasche, FL & Wandelt 2014, arXiv:1409.6308

Samples of the posterior density

- Each sample: a possible "version of the truth"
- The variation between samples quantifies the uncertainty that results from having
  - only one Universe (a more precise version of "cosmic variance")
  - incomplete observations (mask, finite volume and number of galaxies, selection effects)
  - imperfect data (noise, biases…)

see also FL, Pisani & Wandelt, arXiv:1403.1260

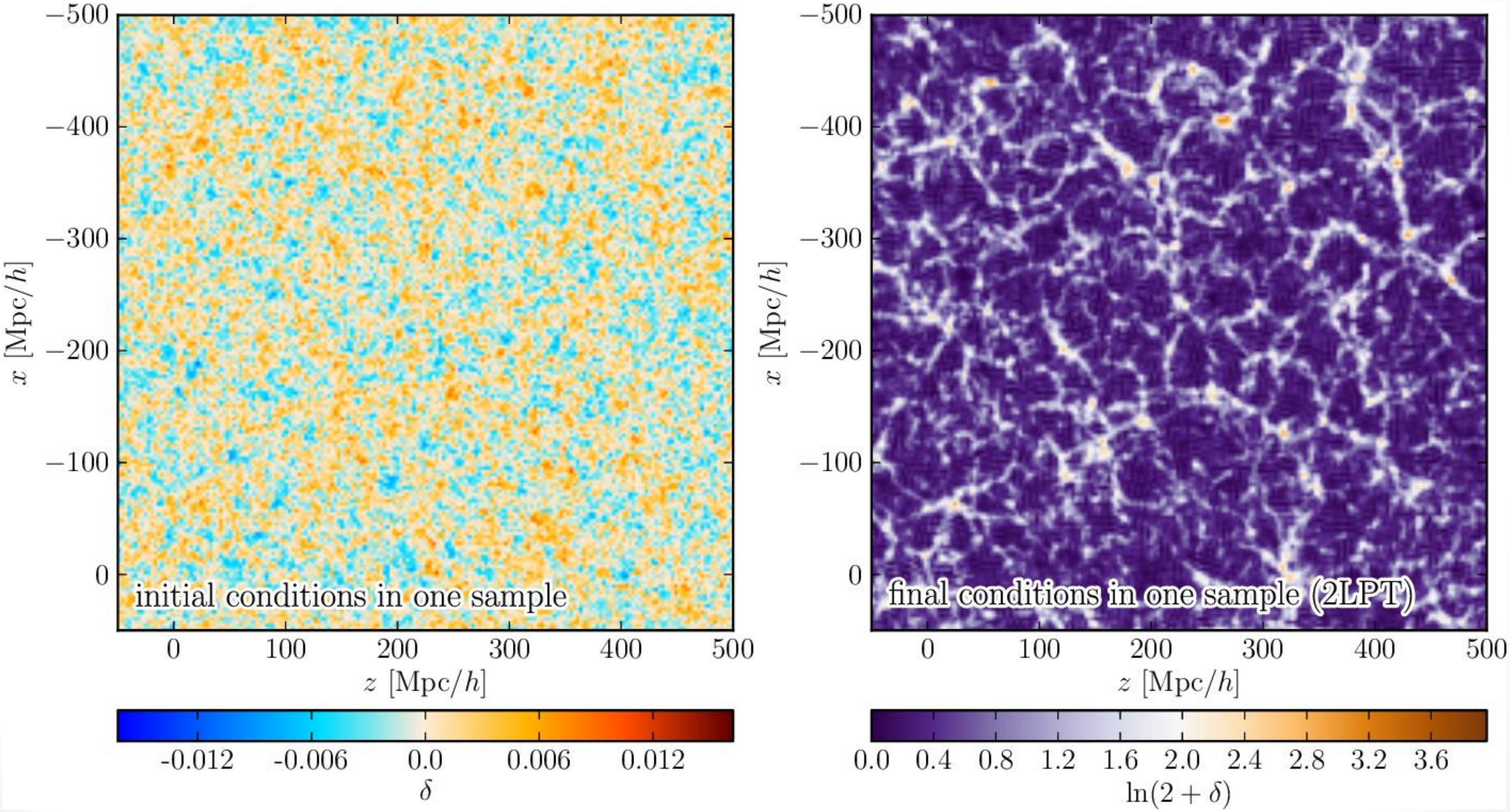
Bayesian chronocosmography from SDSS DR7



Data

Jasche, FL & Wandelt 2014, arXiv:1409.6308

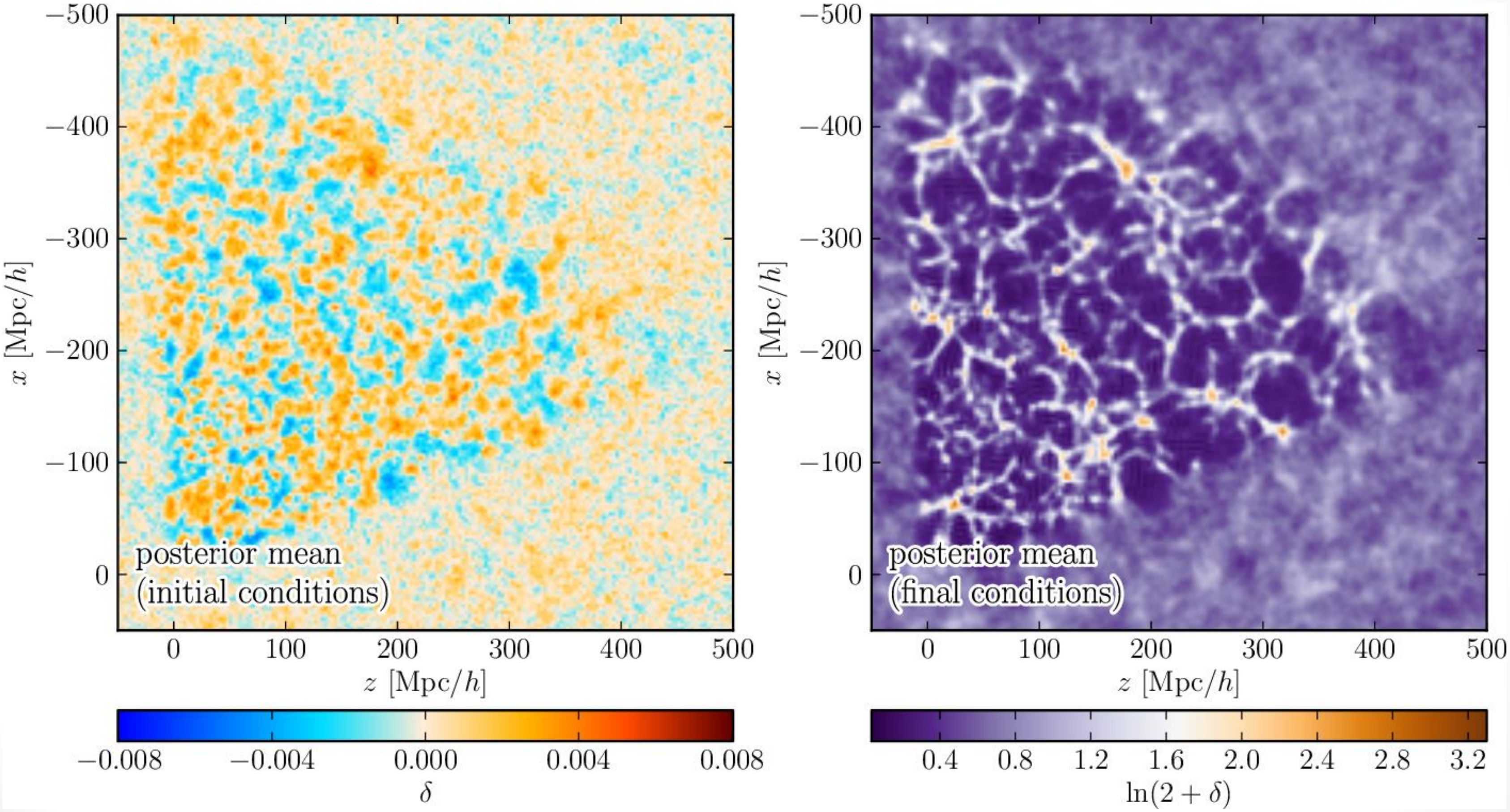
Bayesian chronocosmography from SDSS DR7



One sample

Jasche, FL & Wandelt 2014, arXiv:1409.6308

Bayesian chronocosmography from SDSS DR7



Posterior mean

Jasche, FL & Wandelt 2014, arXiv:1409.6308

3. The Non-Linear Regime of Structure Formation

- Non-linear filtering of BORG results
- Remapping Lagrangian Perturbation Theory
- The COLA method

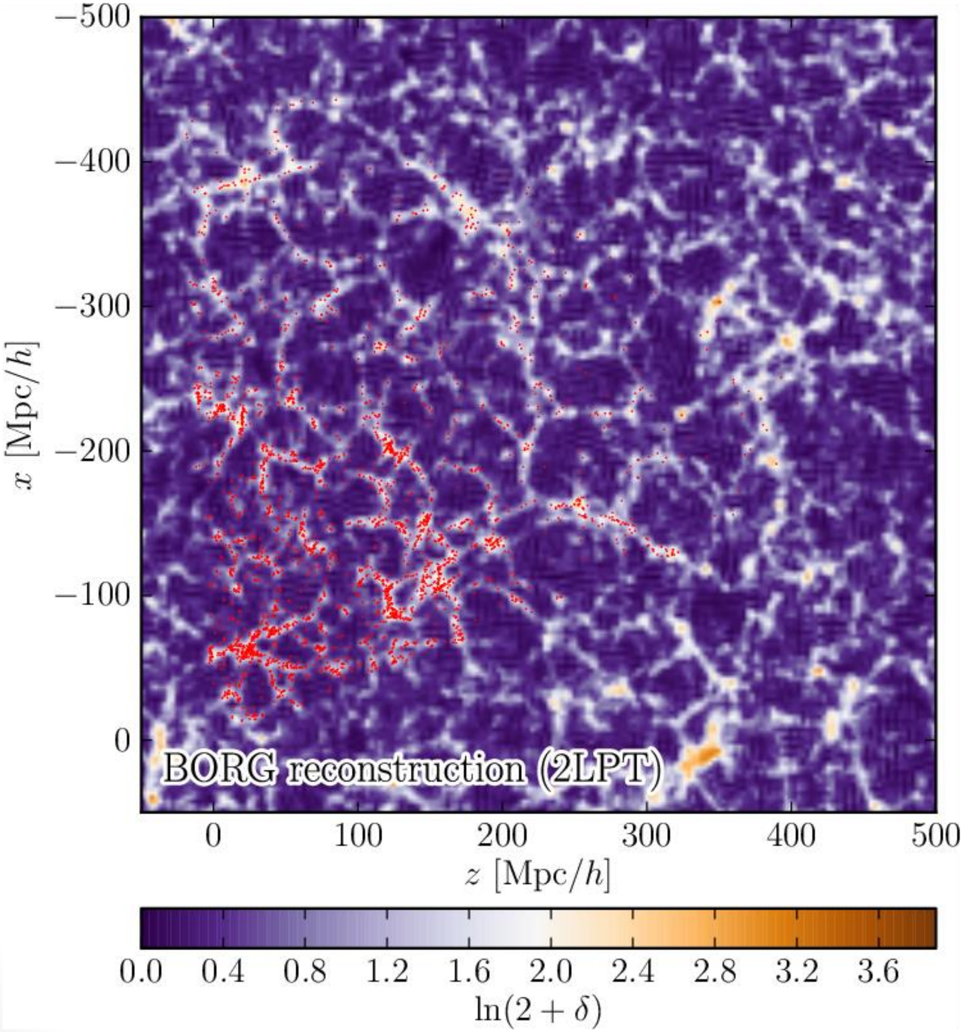
F. Leclercq, J. Jasche, P. M. Sutter, N. Hamaus, B. Wandelt, arXiv:1410.0355.
Dark matter voids in the SDSS galaxy survey

F. Leclercq, J. Jasche, H. Gil-Marín, B. Wandelt, arXiv:1305.4642.
One-point remapping of Lagrangian perturbation theory in the mildly non-linear regime of cosmic structure formation

S. Tassev, M. Zaldarriaga, D. Eisenstein, arXiv:1301.0322.
Solving Large Scale Structure in Ten Easy Steps with COLA

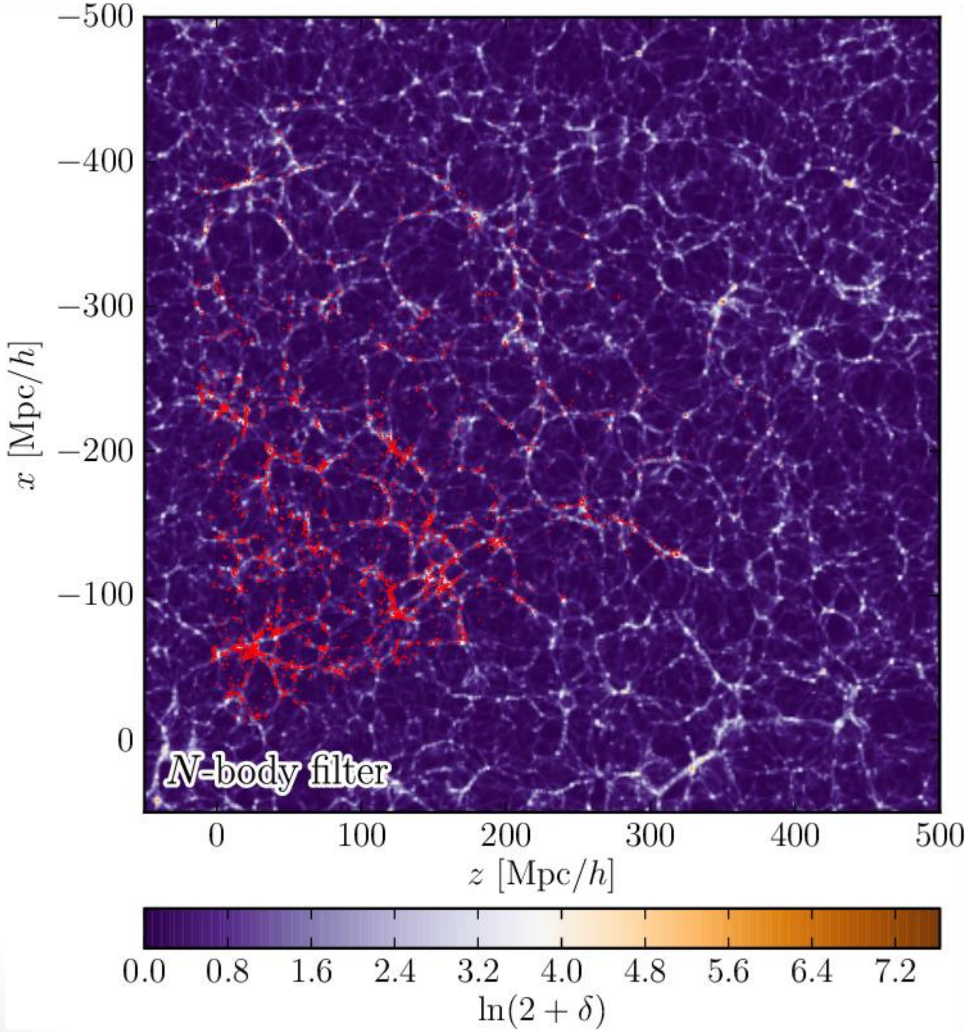
S. Tassev, D. Eisenstein, B. Wandelt, M. Zaldarriaga, in prep. + F. Leclercq, B. Wandelt, *et al.*, in prep.
Extending the N-body Comoving Lagrangian Acceleration Method to the Spatial Domain

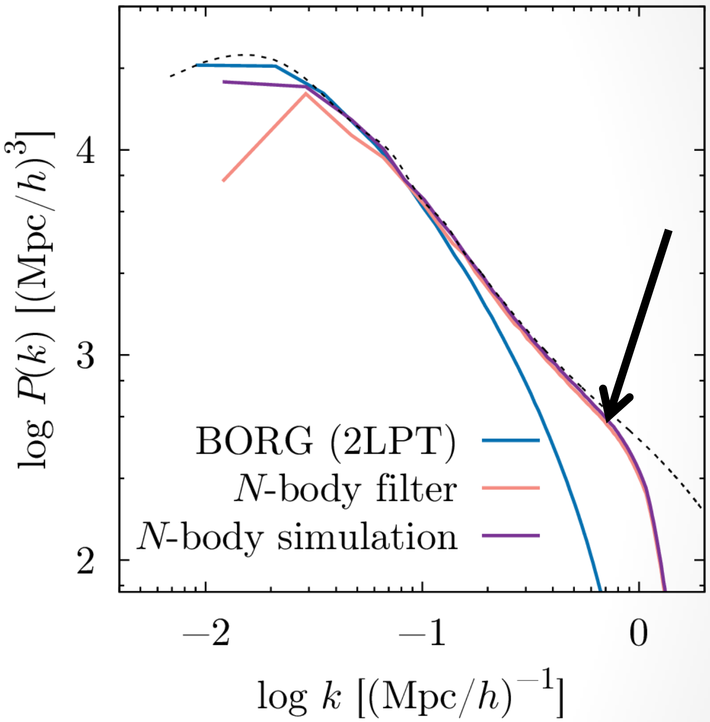
Non-linear filtering



FL, Jasche, Sutter, Hamaus & Wandelt 2014, arXiv:1410.0355 + Jasche, FL, Romano-Diaz & Wandelt, in prep.

Non-linear filtering

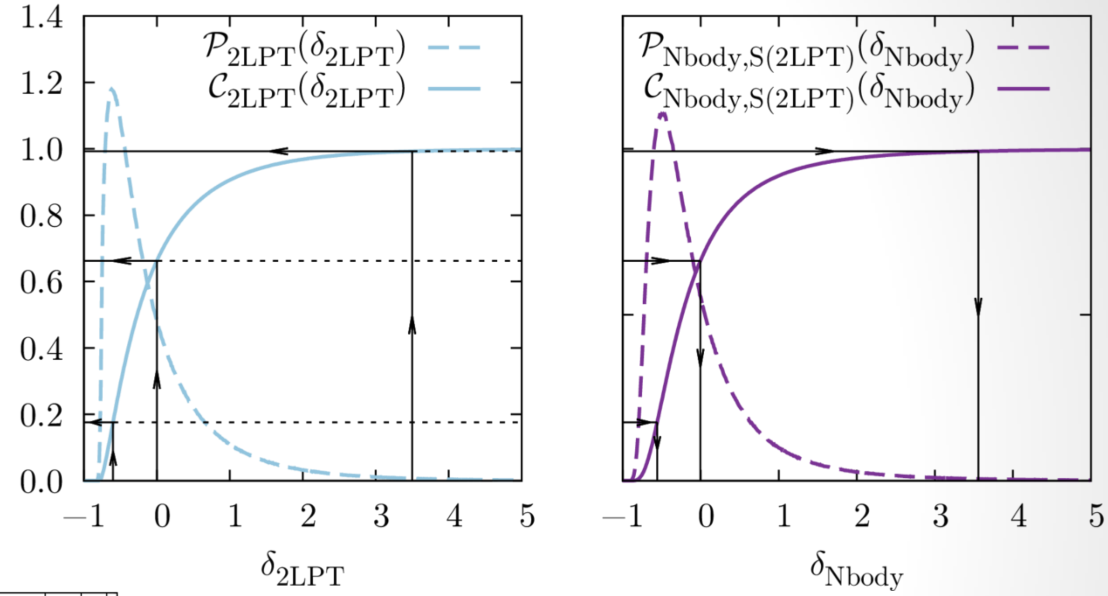




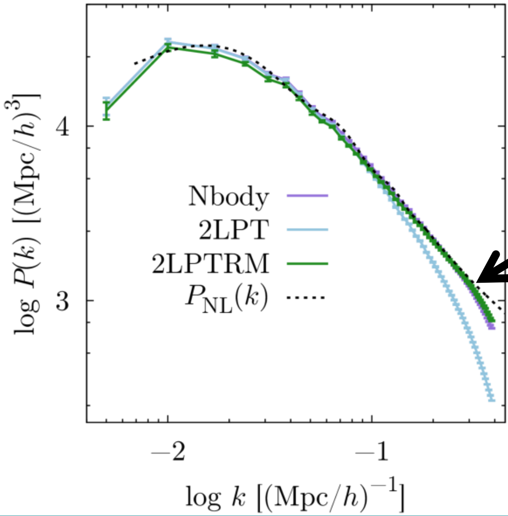
FL, Jasche, Sutter, Hamaus & Wandelt 2014, arXiv:1410.0355 + Jasche, FL, Romano-Diaz & Wandelt, in prep.

Remapping 2LPT in the mildly non-linear regime

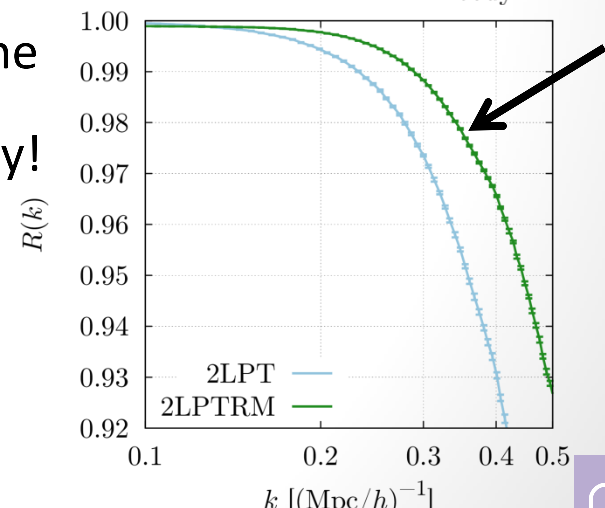
FL, Jasche, Gil-Marín & Wandelt 2013, arXiv:1305.4642



- Replacing the one-point distribution of 2LPT by one which accounts for the full non-linear system…
- …also improves the higher-order correlators…



- …and the phase accuracy!



COLA: *COmoving Lagrangian Acceleration*

- Write the displacement vector as: $\mathbf{s} = \mathbf{s}_{\mathrm{LPT}} + \mathbf{s}_{\mathrm{MC}}$

Tassev & Zaldarriaga 2012, arXiv:1203.5785

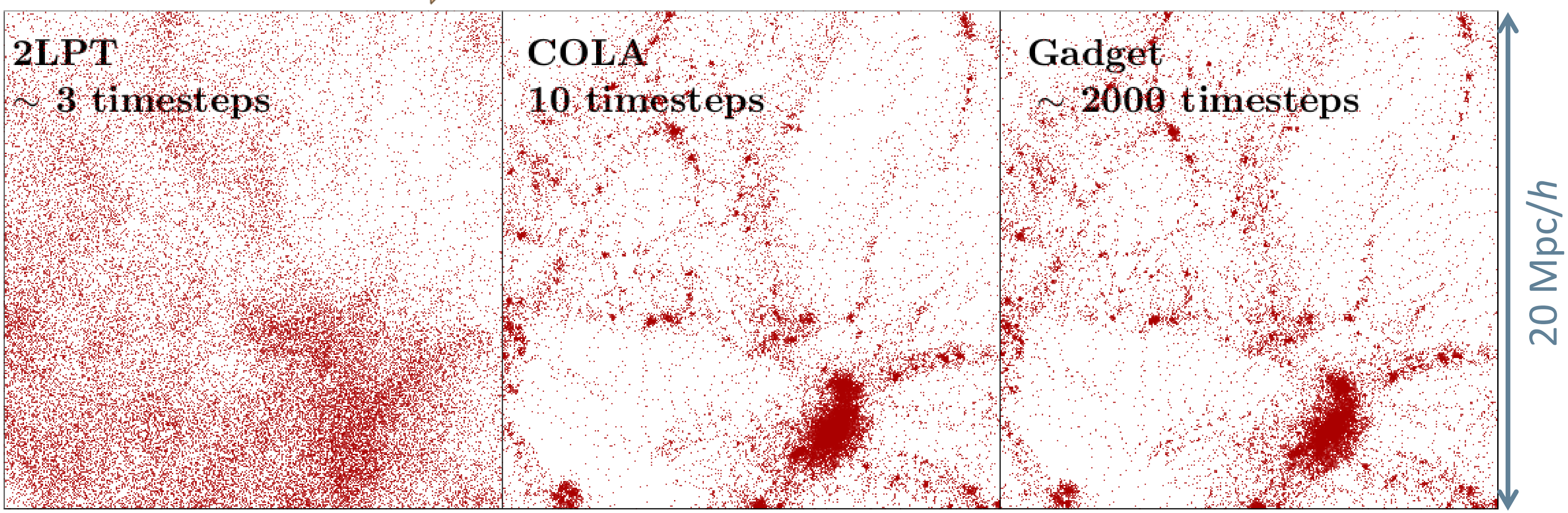
- Time-stepping (omitted constants and Hubble expansion):

Standard:

$$\partial_\tau^2 \mathbf{s} = -\nabla\Phi$$

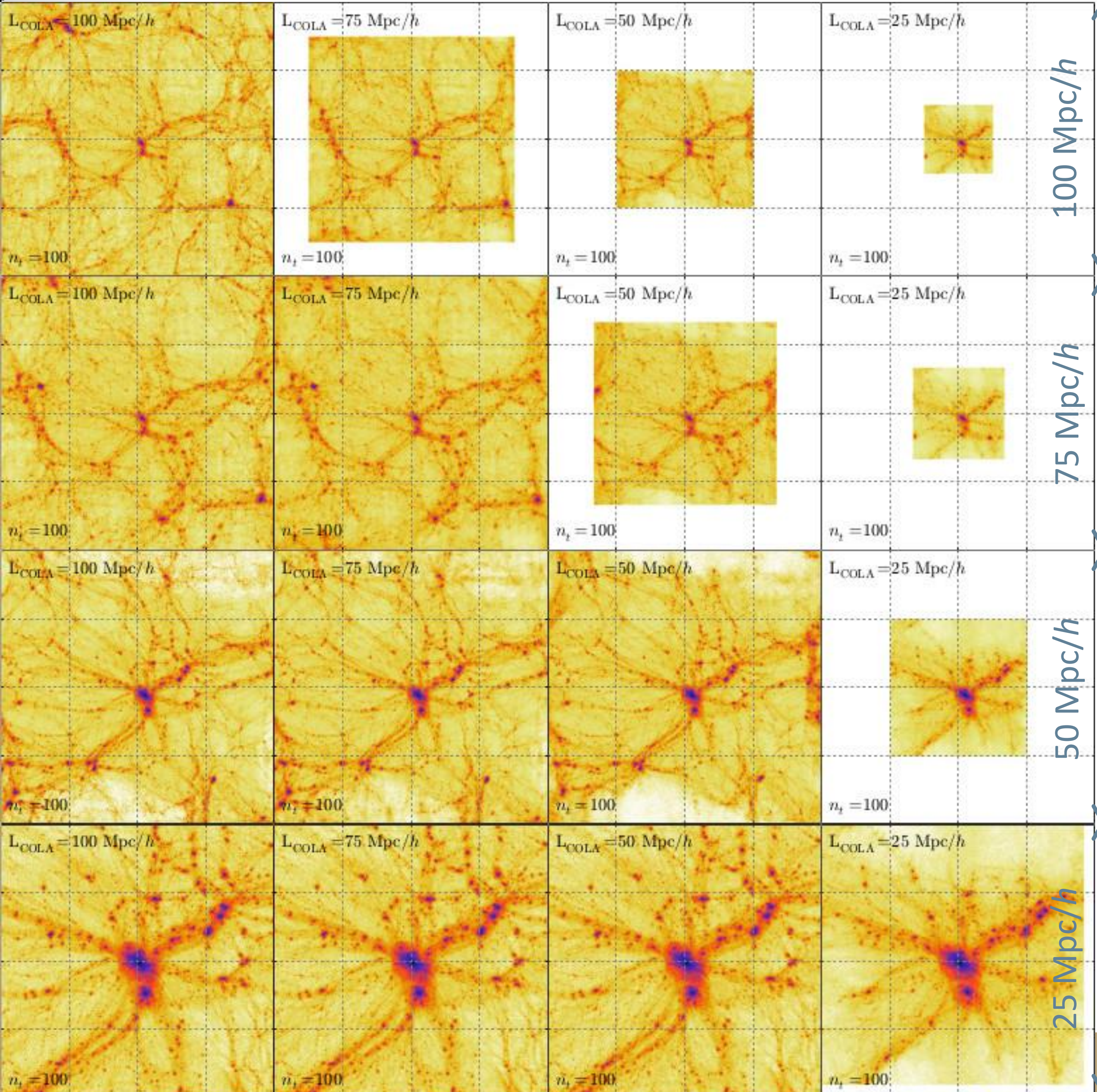
Modified:

$$\partial_\tau^2 \mathbf{s}_{\mathrm{MC}} = \partial_\tau^2(\mathbf{s} - \mathbf{s}_{\mathrm{LPT}}) = -\nabla\Phi - \partial_\tau^2 \mathbf{s}_{\mathrm{LPT}}$$



Original COLA "in time"

Tassev, Zaldarriaga & Einsenstein 2013, arXiv:1301.0322



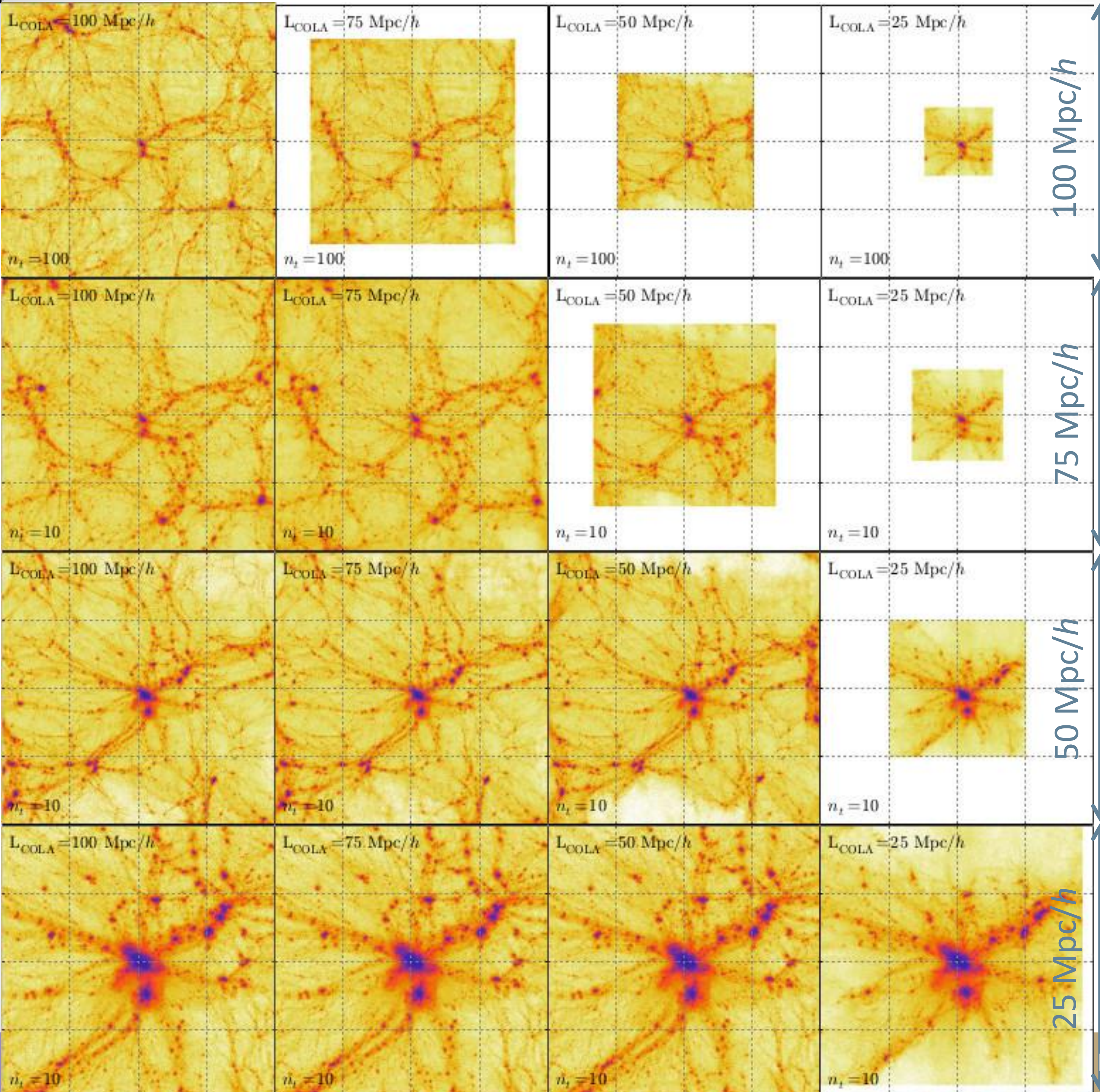
Extending COLA

New COLA "in space"

$$n_t = 100$$

Tassev, Eisenstein, Wandelt & Zaldarriaga, in prep.

+ FL, Wandelt, *et al.*, in prep.



Extending COLA

New COLA "in space and time"

$$n_t = 10$$

Tassev, Eisenstein, Wandelt & Zaldarriaga, in prep.

+ FL, Wandelt, *et al.*, in prep.

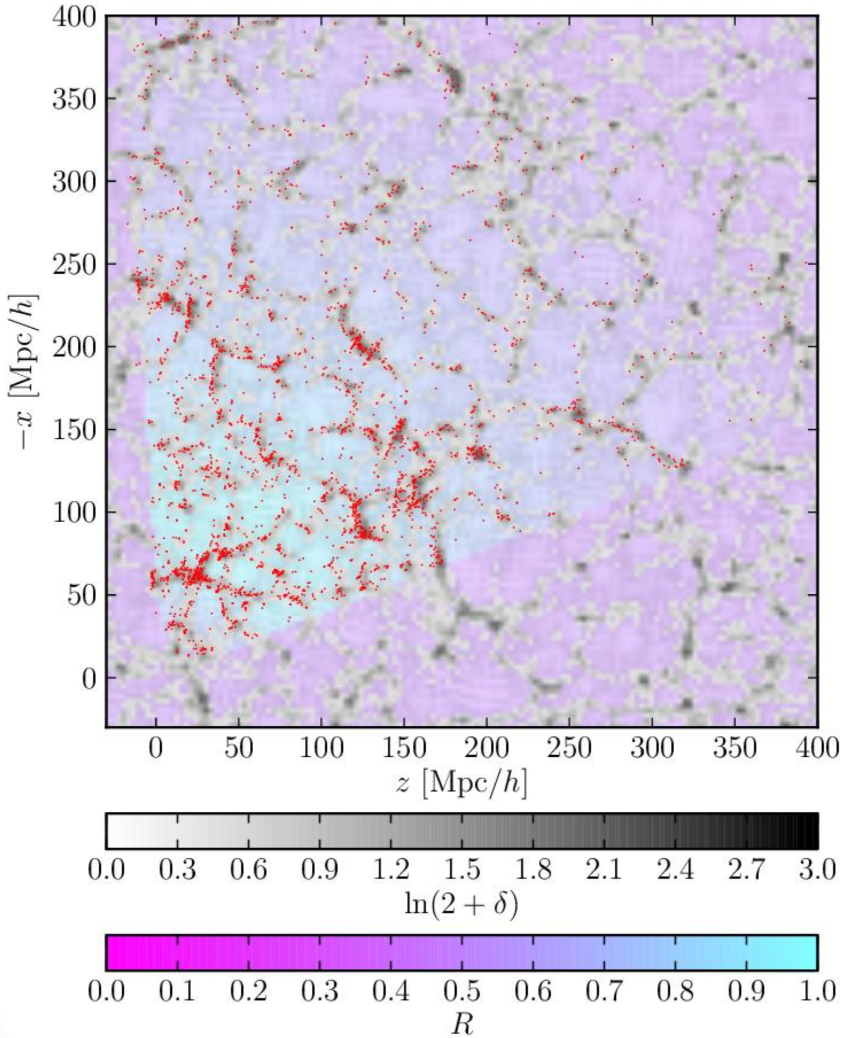
4. Cosmic Web Classification

- Dark matter voids in the SDSS
- Tidal shear analysis in the SDSS, dynamic structure type classification

F. Leclercq, J. Jasche, P. M. Sutter, N. Hamaus, B. Wandelt, arXiv:1410.0355.
Dark matter voids in the SDSS galaxy survey

F. Leclercq, J. Jasche, B. Wandelt, in prep.
Bayesian analysis of the dynamic cosmic web in the SDSS galaxy survey

Dark matter voids in the SDSS



FL, Jasche, Sutter, Hamaus & Wandelt 2014, arXiv:1410.0355

- Why?

Sparsity & **Bias**

Sutter *et al.* 2013, arXiv:1309.5087

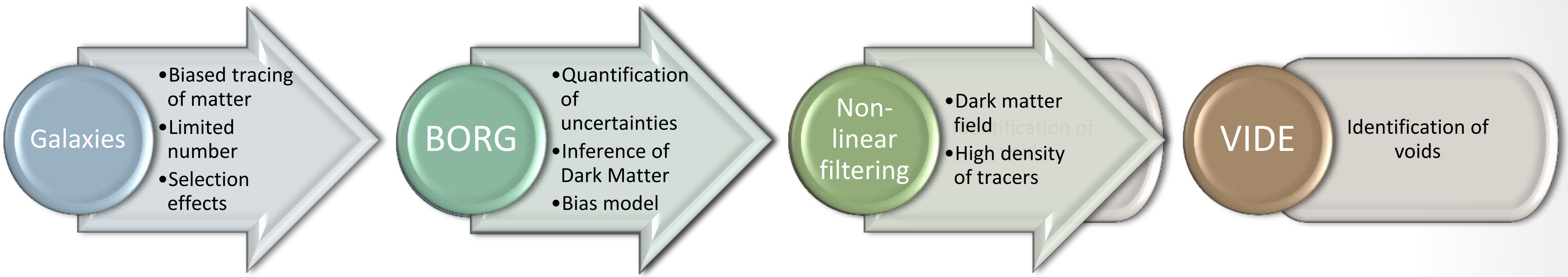
Sutter *et al.* 2013, arXiv:1311.3301

- How?

VIDE toolkit: Sutter *et al.* 2014, arXiv:1406.1191
www.cosmicvoids.net

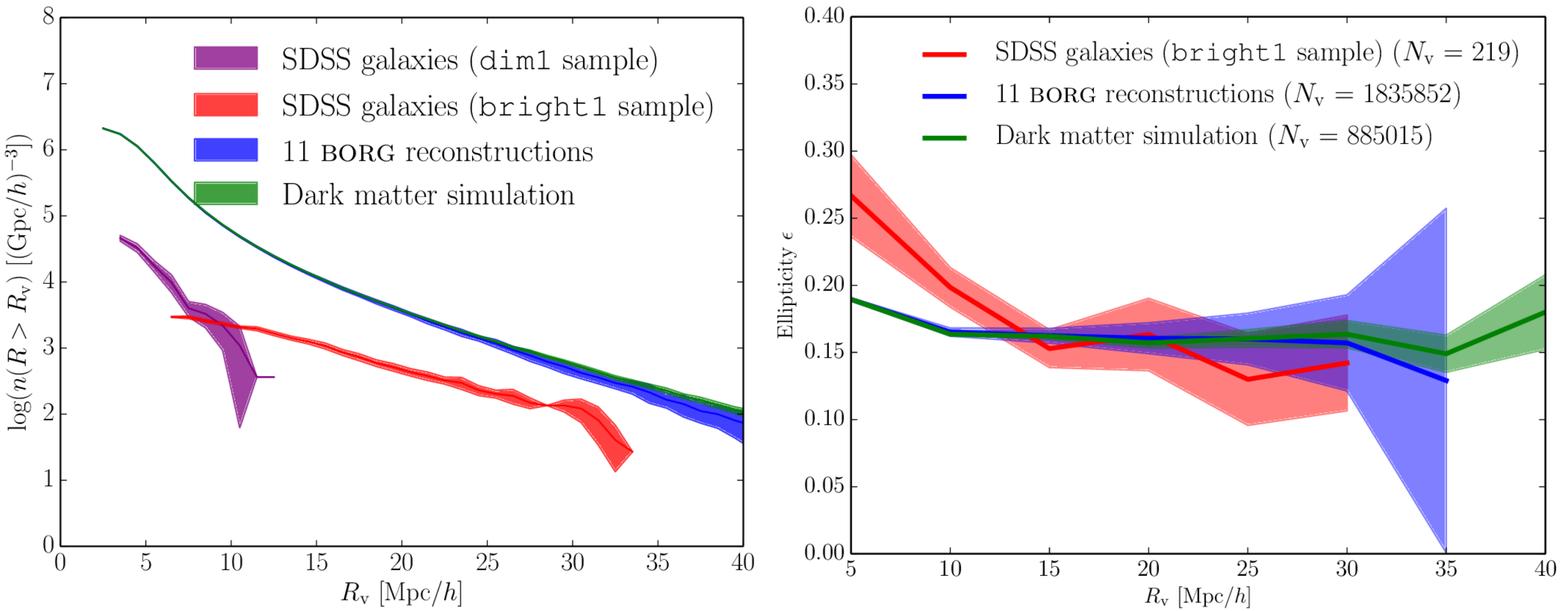
based on ZOBOV: Neyrinck 2007, arXiv:0712.3049

Dark matter voids: pipeline



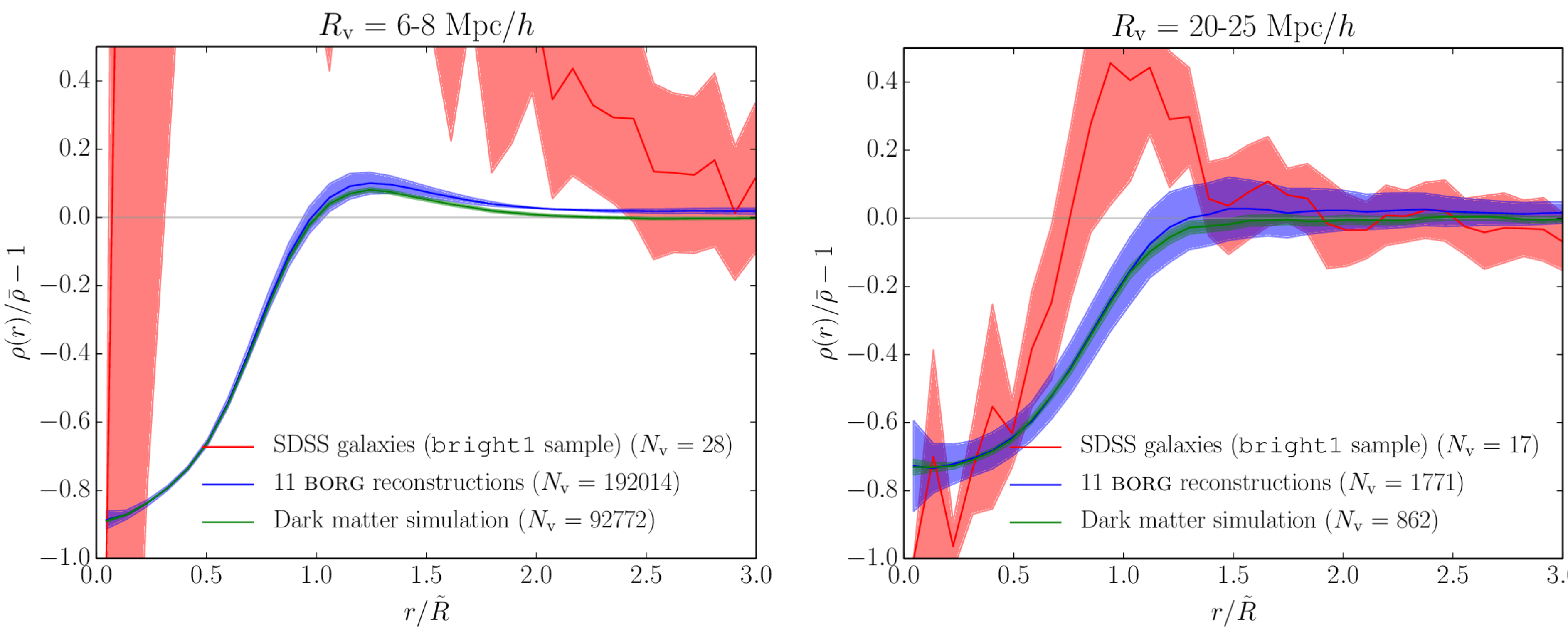
FL, Jasche, Sutter, Hamaus & Wandelt 2014, arXiv:1410.0355

Dark matter void properties



FL, Jasche, Sutter, Hamaus & Wandelt 2014, arXiv:1410.0355

Dark matter void properties



All catalogs will be made publicly available at
www.cosmicvoids.net

FL, Jasche, Sutter, Hamaus & Wandelt 2014, arXiv:1410.0355

Tidal shear analysis

- $\lambda_1, \lambda_2, \lambda_3$: eigenvalues of the tidal field tensor, the Hessian of the gravitational potential: $T_{ij} = \partial_i \partial_j \Phi$
  - Voids: $\lambda_1, \lambda_2, \lambda_3 < 0$
  - Sheets: $\lambda_1 > 0 \text{ and } \lambda_2, \lambda_3 < 0$
  - Filaments: $\lambda_1, \lambda_2 > 0 \text{ and } \lambda_3 < 0$
  - Clusters: $\lambda_1, \lambda_2, \lambda_3 > 0$

Hahn *et al.* 2007, arXiv:astro-ph/0610280

see also:

- Extensions:

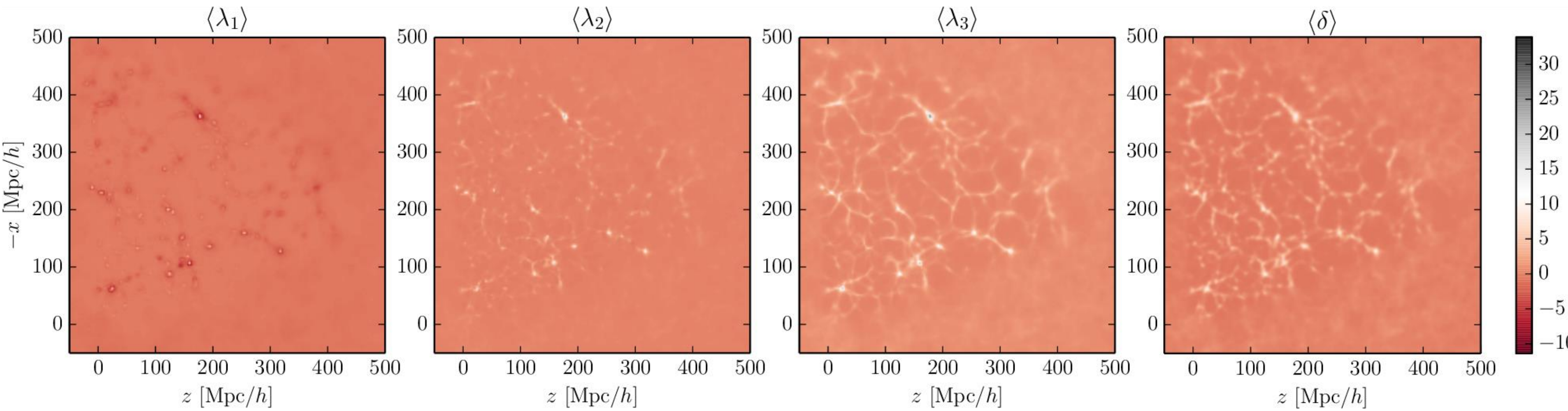
Forero-Romero *et al.* 2008, arXiv:0809.4135
Hoffman *et al.* 2012, arXiv:1201.3367

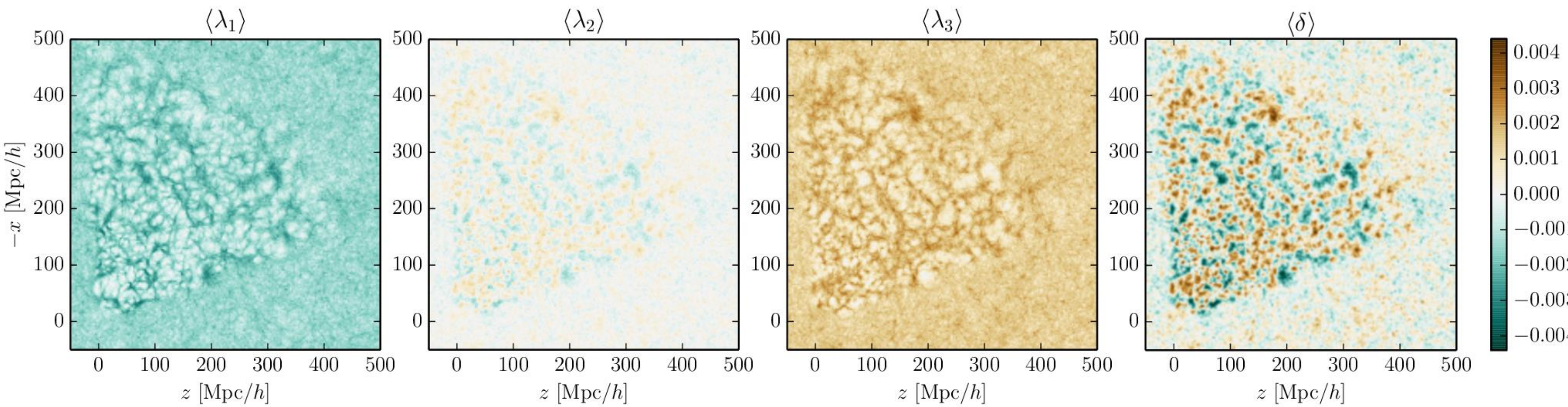
- Similar web classifiers:

DIVA, Lavaux & Wandelt 2010, arXiv:0906.4101
ORIGAMI, Falck, Neyrinck & Szalay 2012, arXiv:1201.2353

Tidal shear analysis

Final conditions



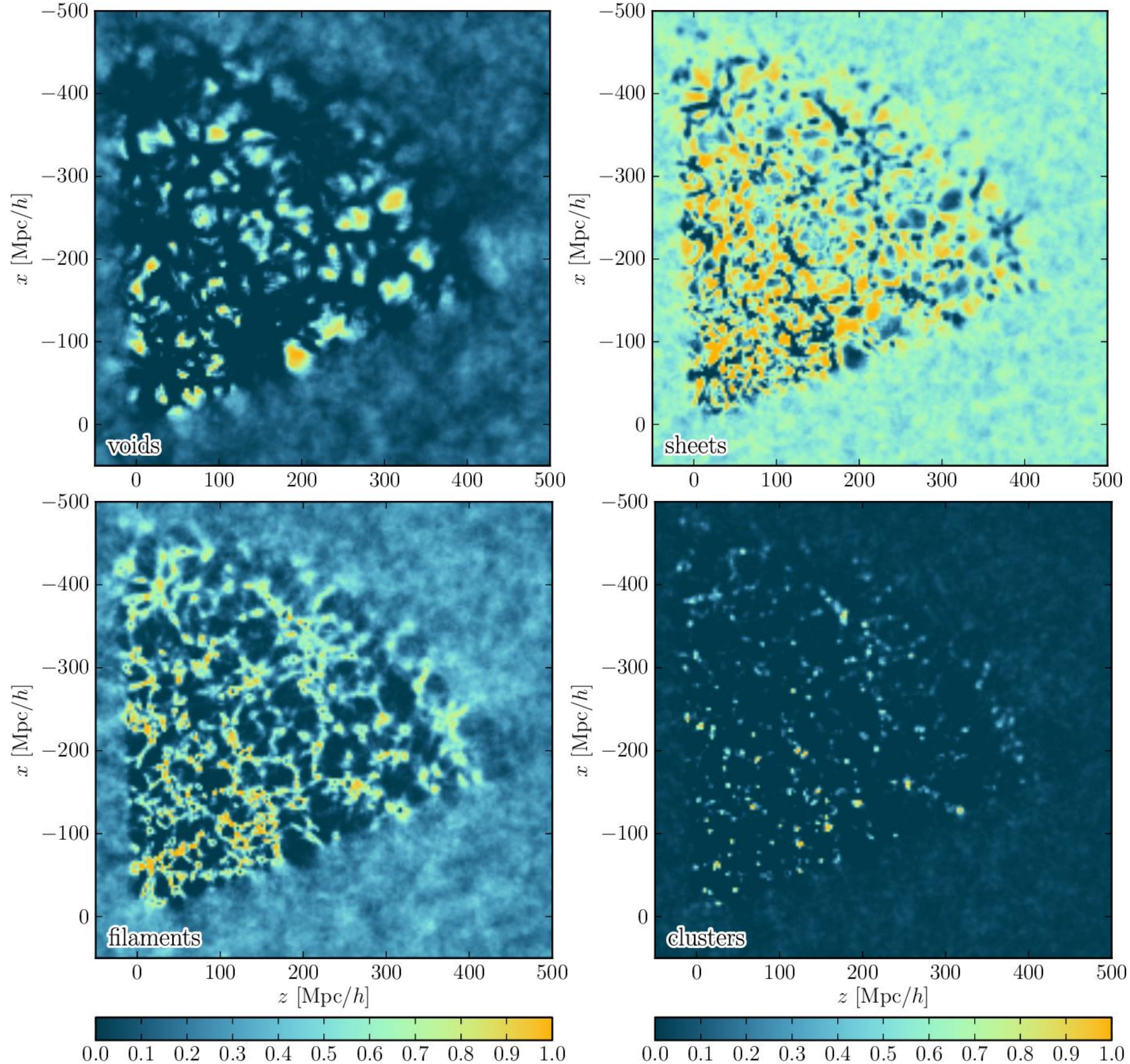


FL, Jasche & Wandelt, in prep.

Initial conditions

Dynamic structures inferred by BORG

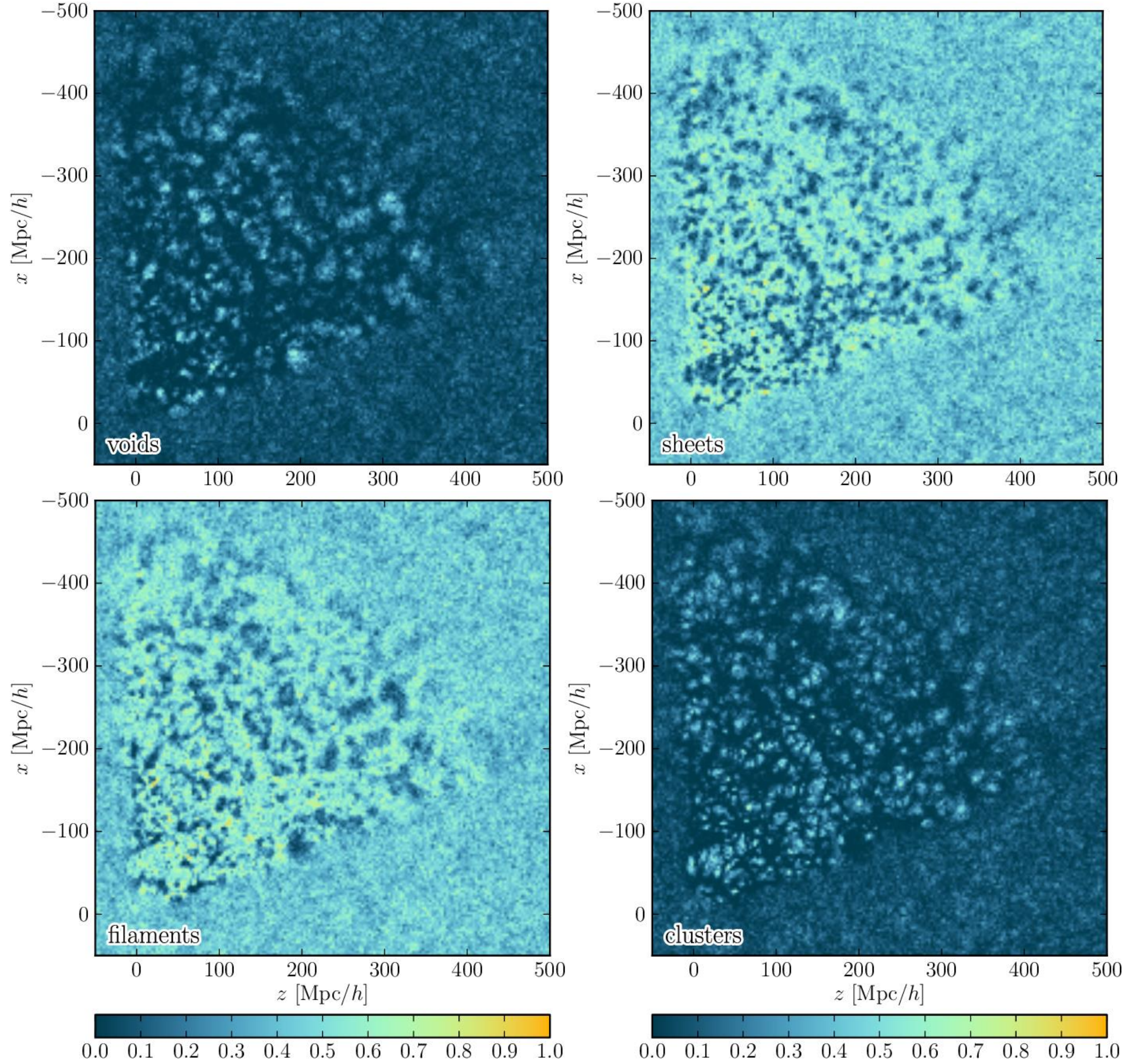
Final conditions



FL, Jasche & Wandelt, in prep. + Chevallard, FL, Jasche & Wandelt, in prep.

Dynamic structures inferred by BORG

Initial conditions

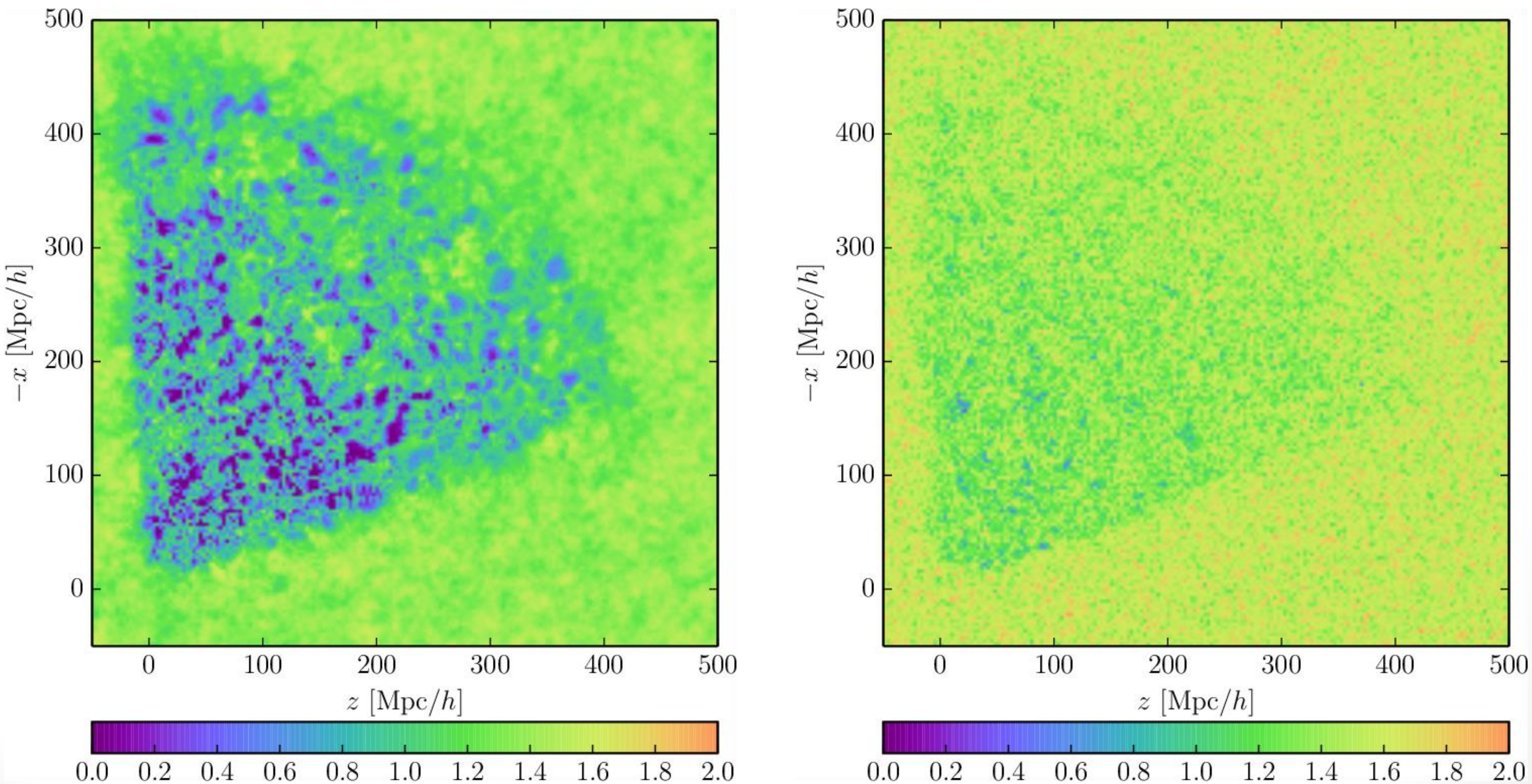


FL, Jasche & Wandelt, in prep. + Chevallard, FL, Jasche & Wandelt, in prep.

Entropy of the structure types pdf

$$H\left[\mathcal{P}(\mathrm{T}(\vec{x}_k)|d)\right] \equiv -\sum_{i=0}^{3} \mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d)\log_2(\mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d))$$ in shannons (Sh)

Final conditions | Initial conditions



FL, Jasche & Wandelt, in prep.

A decision rule for structure classification

- Space of "input features":

$$\{\mathrm{T}_0 = \text{void}, \mathrm{T}_1 = \text{sheet}, \mathrm{T}_2 = \text{filament}, \mathrm{T}_3 = \text{cluster}\}$$

- Space of "actions":

$$\{a_0 = \text{"decide void"}, a_1 = \text{"decide sheet"}, a_2 = \text{"decide filament"}, a_3 = \text{"decide cluster"}, a_{-1} = \text{"do not decide"}\}$$

⇨ A problem of **Bayesian decision theory**:
one should take the action which maximizes the utility

$$U(a_j(\vec{x}_k)|d) = \sum_{i=0}^{3} G(a_j|\mathrm{T}_i)\,\mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d)$$

- How to write down the gain functions?

FL, Jasche & Wandelt, in prep.

Gambling with the Universe

- One proposal:

$$G(a_j|\mathrm{T}_i) = \begin{cases} \frac{1}{\mathcal{P}(\mathrm{T}_i)} - \alpha & \text{if } j \in [\![0,3]\!] \text{ and } i = j \\ -\alpha & \text{if } j \in [\![0,3]\!] \text{ and } i \neq j \\ 0 & \text{if } j = -1. \end{cases}$$

"Winning"

"Loosing"

"Not playing"

- Without data, the expected utility is

$$U(a_j) = 1 - \alpha \quad \text{if } j \neq 1$$

"Playing the game"

$$U(a_{-1}) = 0$$

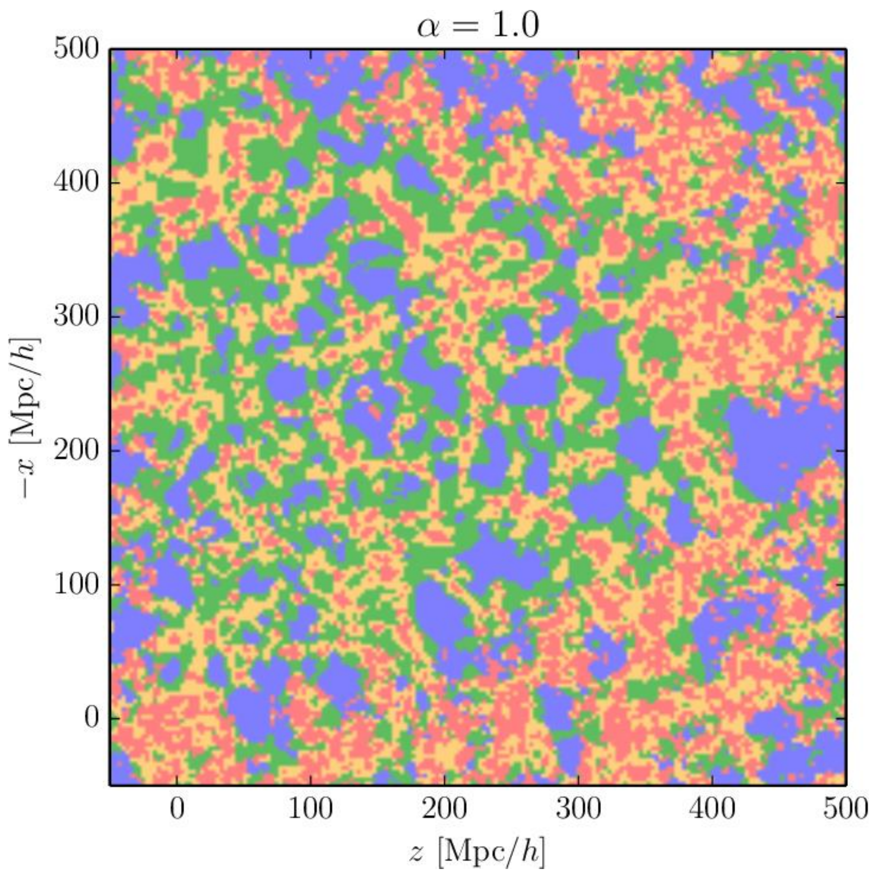
"Not playing the game"

- With $\alpha = 1$, it's a *fair game* ⇒ always play ⇒ "speculative map" of the LSS
- Values $\alpha > 1$ represent an *aversion for risk* ⇒ increasingly "conservative maps" of the LSS

FL, Jasche & Wandelt, in prep.

Playing the game…

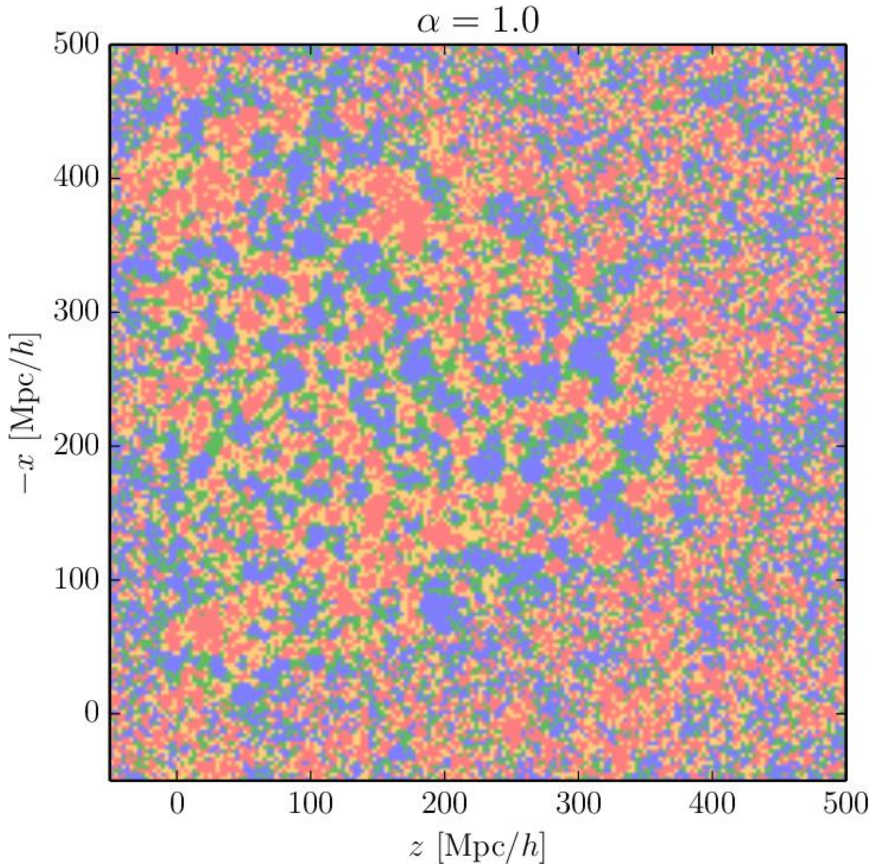
Final conditions



FL, Jasche & Wandelt, in prep.

Playing the game…

Initial conditions



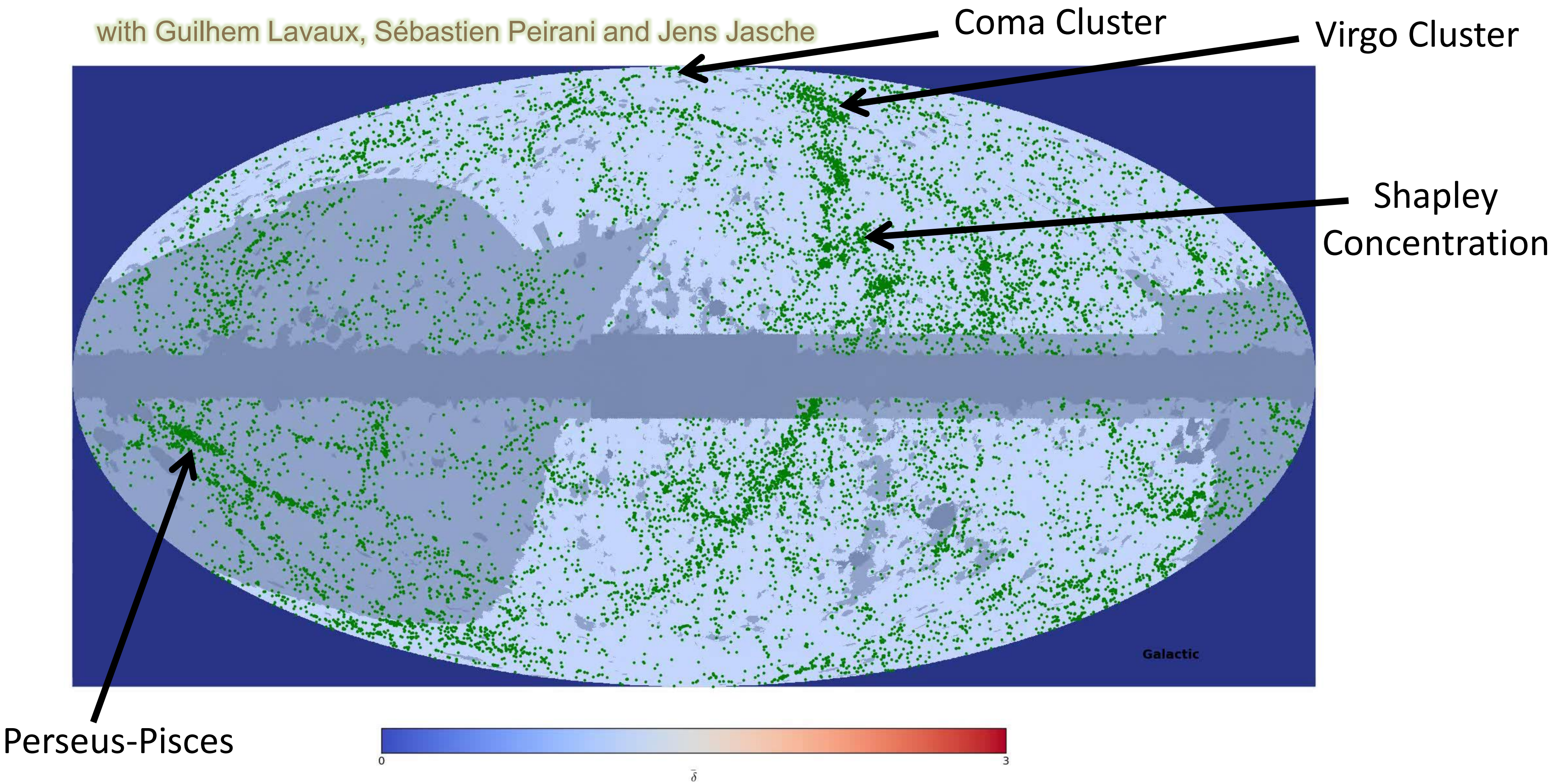
FL, Jasche & Wandelt, in prep.

5. The Future

- Constrained simulations of the Local Universe
- Templates as hypothesis generators

Ongoing project: PLUS: *the Paris Local Universe Simulation*

with Guilhem Lavaux, Sébastien Peirani and Jens Jasche

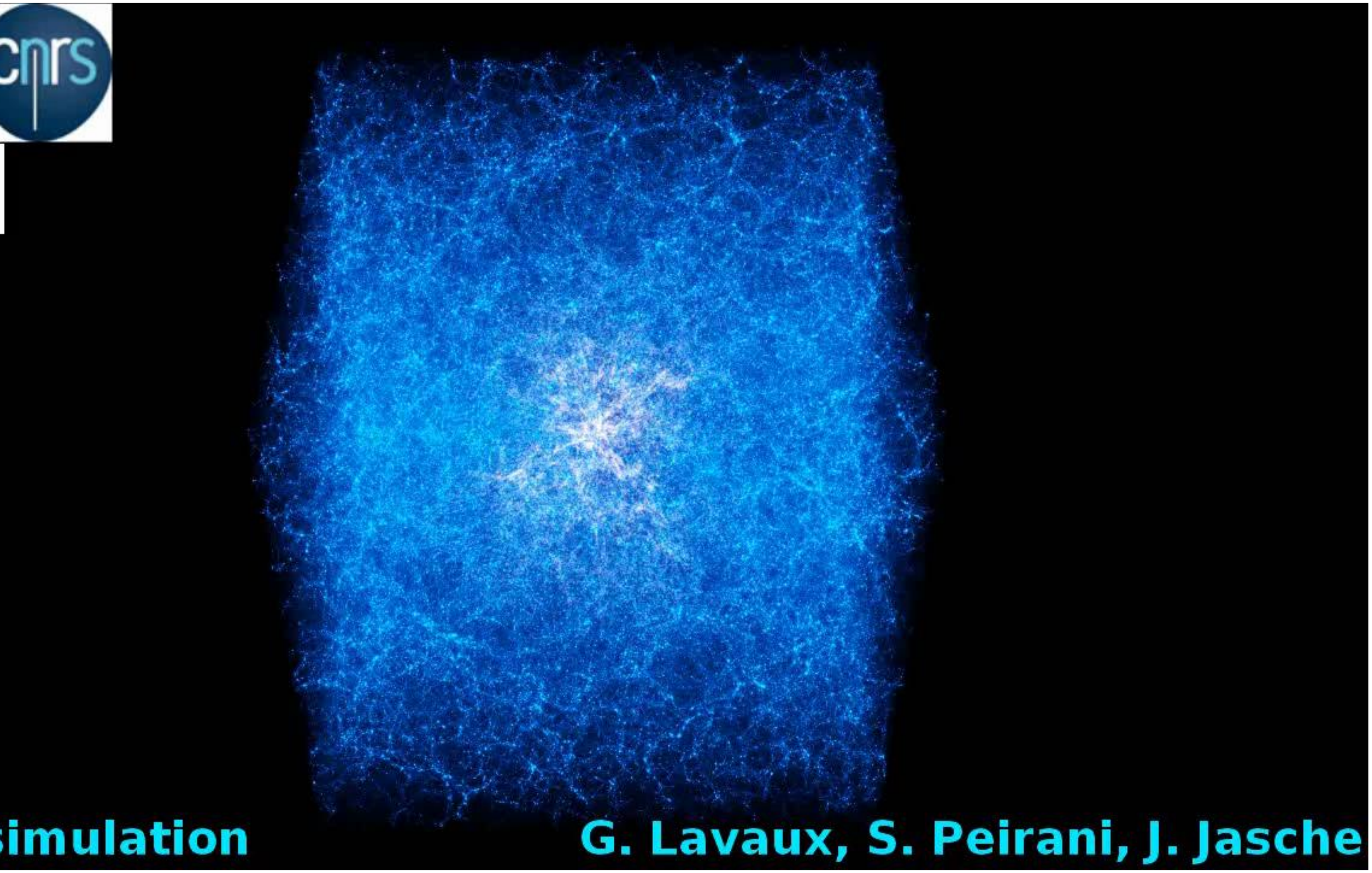


600 Mpc/*h* box, 60 Mpc/*h* projection, 512^3 dark matter particles

2M++ catalog: Lavaux & Hudson 2011, arXiv:1105.6107 (compiled 2MASS, 6dF, SDSS DR7)

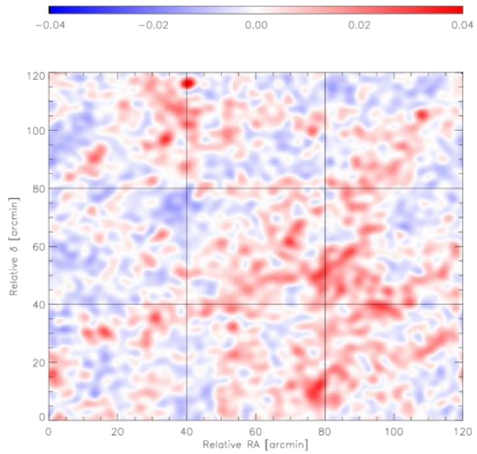
Ongoing project: PLUS: *the Paris Local Universe Simulation*

with Guilhem Lavaux, Sébastien Peirani and Jens Jasche



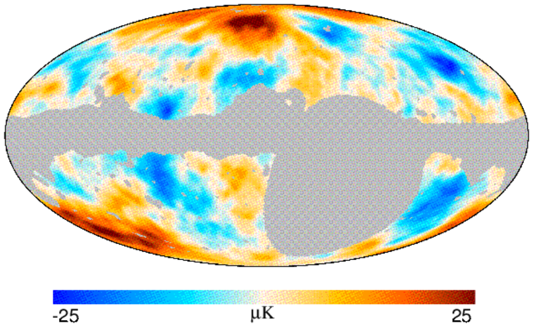
The future: templates as hypothesis generators

Weak Lensing



Szepietowski *et al*. 2013, arXiv:1306.5324

Integrated Sachs-Wolfe effect



Planck collaboration 2013 XIX, arXiv:1303.5079

Ho, Hirata, Padmanabhan, Seljak & Bahcall 2008, arXiv:0801.0642

Summary & Conclusions

- Bayesian large-scale structure inference in 10 millions dimensions is possible!
    - Uncertainty quantification (noise, survey geometry, selection effects and biases)
    - Non-linear and non-Gaussian inference with improving techniques
- Application to data: four-dimensional chronocosmography
    - Simultaneous analysis of the morphology and formation history of the large-scale structure
    - Physical reconstruction of the initial conditions
    - Inference of cosmic voids at the level of the dark matter distribution
    - Characterization of the dynamic cosmic web underlying galaxies