Bayesian analyses of galaxy surveys

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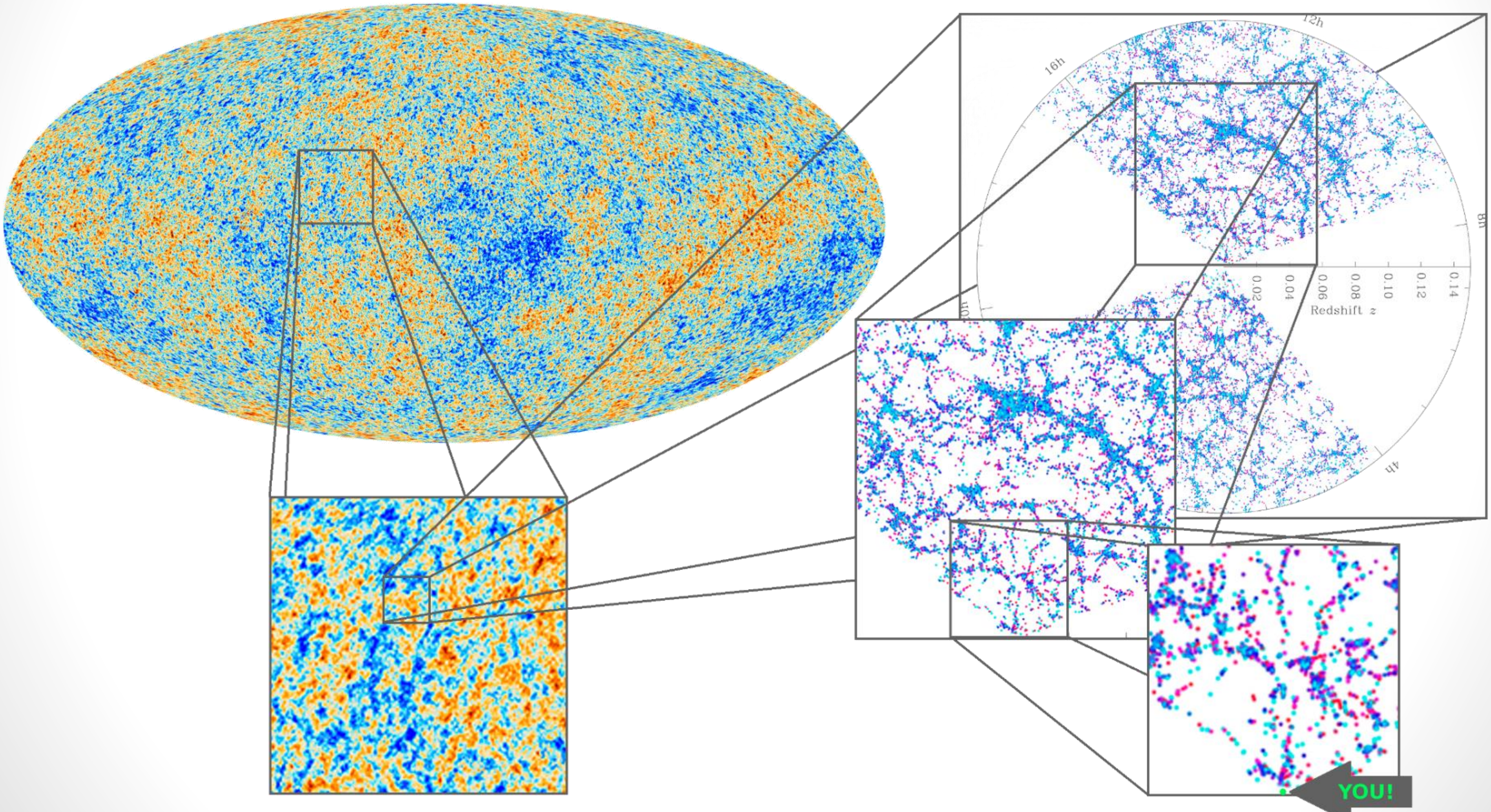
The big picture: the Universe is highly structured

You are here. Make the best of it…



Planck collaboration (2013-2015)

M. Blanton and the Sloan Digital Sky Survey (2010-2013)

What we want to know from the large-scale structure

The LSS is a vast source of knowledge:

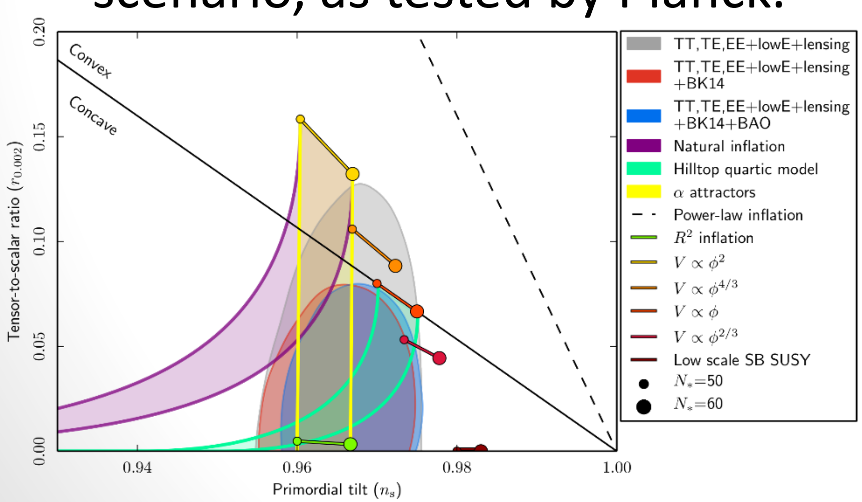
- Cosmology:
  - ΛCDM : cosmological parameters and tests against alternatives,
  - Physical nature of the dark components,
  - Neutrinos : number and masses,
  - Geometry of the Universe,
  - Tests of General Relativity,
  - Initial conditions and link to high energy physics
- Astrophysics: galaxy formation and evolution as a function of their environment
  - Galaxy properties (colours, chemical composition, shapes),
  - Intrinsic alignments, intrinsic size-magnitude correlations

We have theoretical and computer models…

- **Initial conditions:**
  a Gaussian random field

$$\mathcal{P}(\delta^{\mathrm{i}}|S) = \frac{1}{\sqrt{|2\pi S|}} \exp\left(-\frac{1}{2}\sum_{x,x'} \delta^{\mathrm{i}}_x S^{-1}_{xx'} \delta^{\mathrm{i}}_{x'}\right)$$

Everything seems consistent with the simplest inflationary scenario, as tested by Planck.

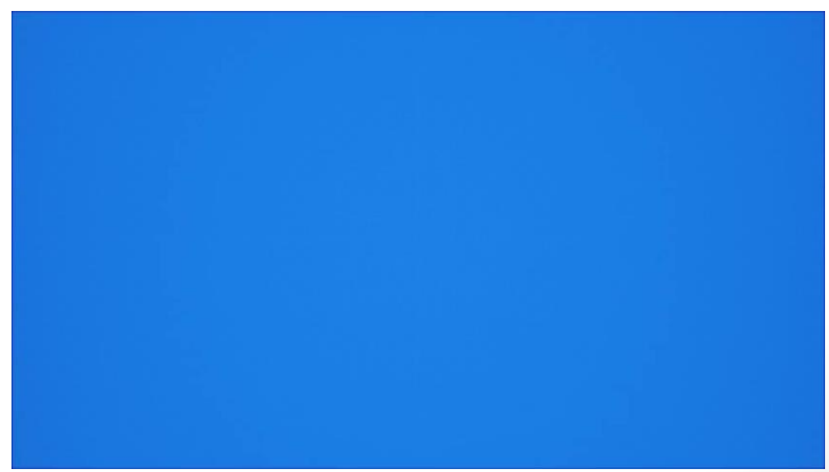


Planck 2018 X, 1807.06211

- **Structure formation:**
  numerical solution of the Vlasov-Poisson system for dark matter dynamics

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{ma} \cdot \nabla f - ma\nabla\Phi \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$

$$\Delta\Phi = 4\pi \mathrm{G} a^2 \bar{\rho}\delta$$



Y. Dubois & S. Colombi (IAP)

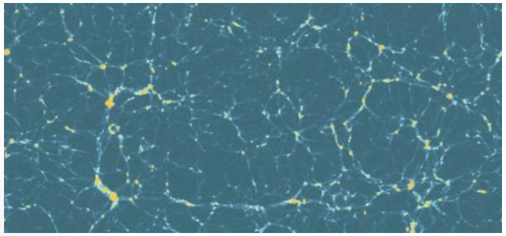
… how do we test these models against survey data?

J. Cham – PhD comics

Redshift range	Volume (Gpc^3)	k_{max} $(Mpc/h)^{-1}$	N_{modes}
0-1	50	0.15	10^7
1-2	140	0.5	$5x10^8$
2-3	160	1.3	10^{10}

M. Zaldarriaga

- Precise tests require many modes.
- In 3D galaxy surveys, the number of modes usable scales as k_{max}^3.



- The challenge: non-linear evolution at small scales and late times.
- The strategy:
  - Pushing down the smallest scale usable for cosmological analysis
  - Using a numerical model linking initial and final conditions

In other words: go beyond the linear and static analysis of the LSS.

Why Bayesian inference?

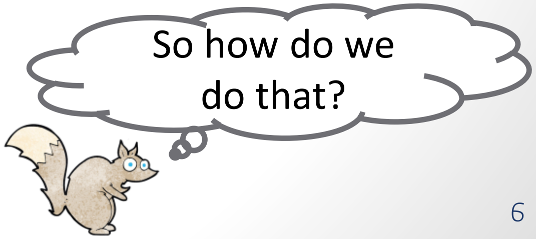
- Inference of signals = ill-posed problem
  - Incomplete observations: finite resolution, survey geometry, selection effects
  - Noise, biases, systematic effects
  - Cosmic variance

➡ No unique recovery is possible!

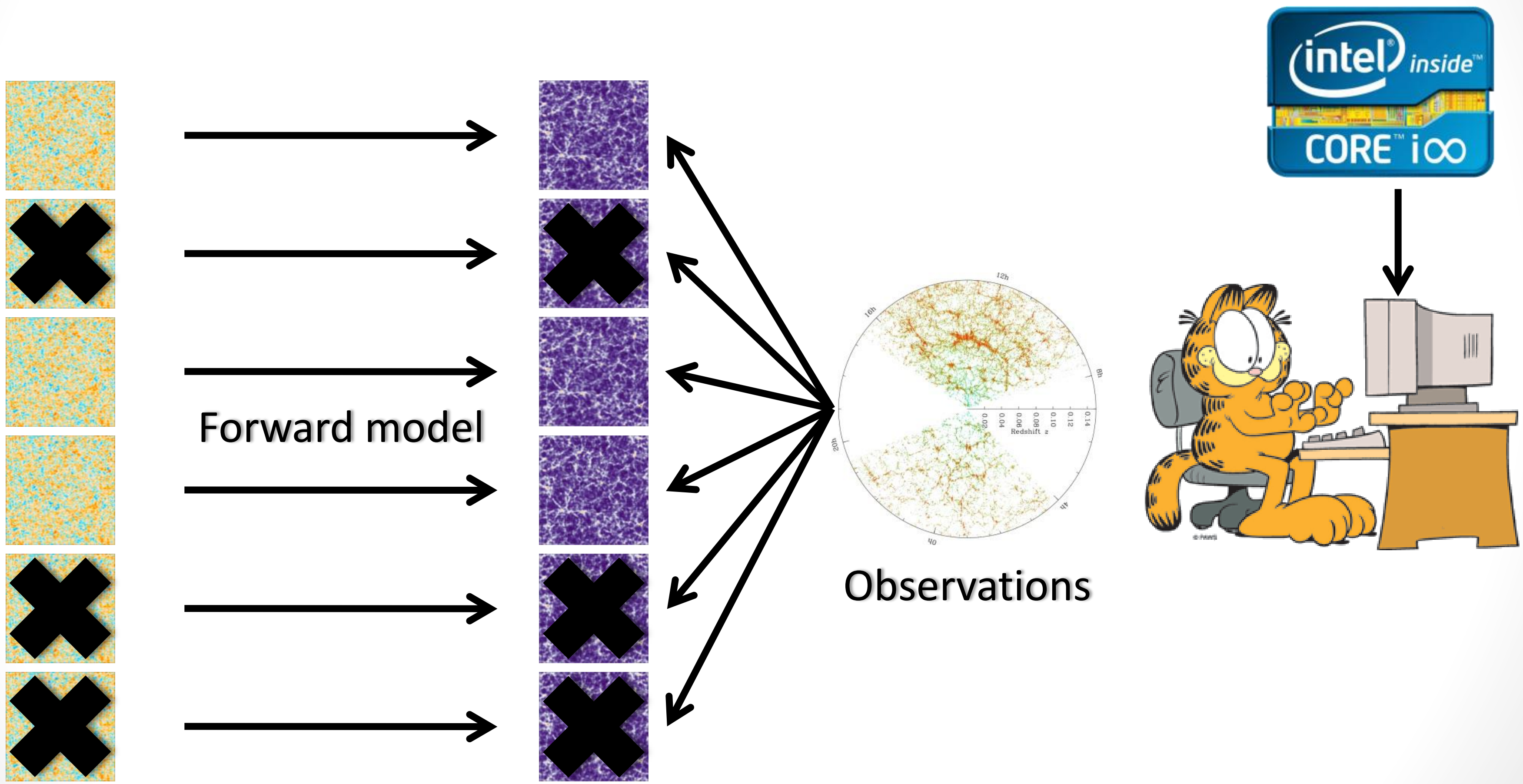
"What is the formation history of the Universe?" ➡ "What is the probability distribution of possible formation histories (signals) compatible with the observations?"

Bayes' theorem: $$\mathcal{P}(s|d)\mathcal{P}(d) = \mathcal{P}(d|s)\mathcal{P}(s)$$

- Cox-Jaynes theorem: Any system to manipulate "*plausibilities*", consistent with Cox's desiderata, is isomorphic to (Bayesian) probability theory



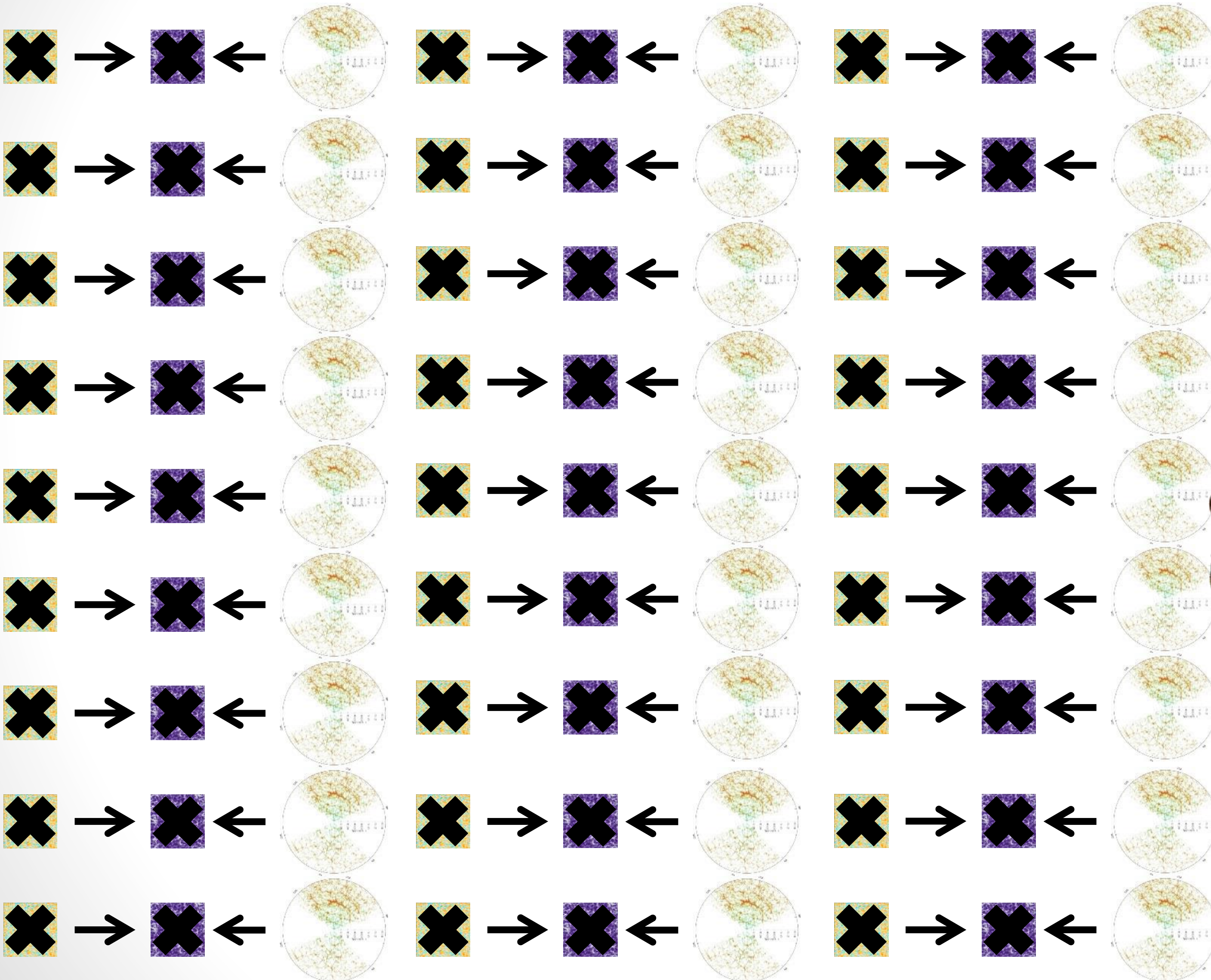
Bayesian forward modelling: the ideal scenario



All possible ICs

All possible FCs

Bayesian forward modelling: the challenge

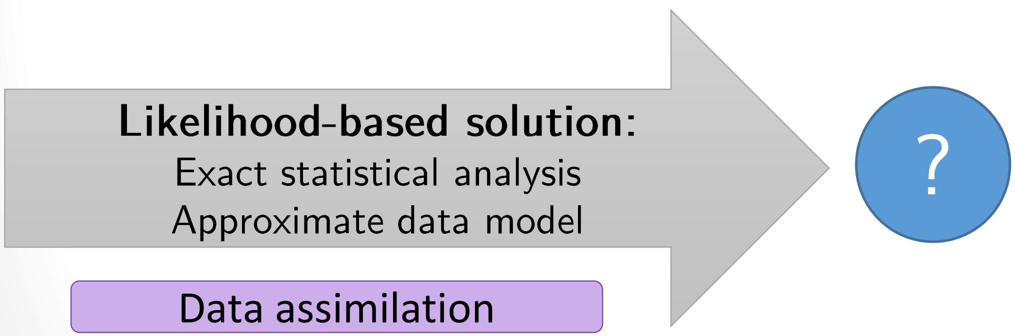


The (true) likelihood lives in

$d \approx 10^7$!

Likelihood-based solution: BORG

Bayesian Origin Reconstruction from Galaxies



Hamiltonian (Hybrid) Monte Carlo

- Use classical mechanics to solve statistical problems!
  - The potential: $\psi(\mathbf{x}) \equiv -\ln p(\mathbf{x})$
  - The Hamiltonian: $H(\mathbf{x},\mathbf{p}) \equiv \frac{1}{2}\mathbf{p}^\intercal \mathbf{M}^{-1}\mathbf{p} + \psi(\mathbf{x})$

$$(\mathbf{x},\mathbf{p}) \Rightarrow \left\{ \begin{aligned} \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} &= \frac{\partial H}{\partial \mathbf{p}} = \mathbf{M}^{-1}\mathbf{p} \\ \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} &= -\frac{\partial H}{\partial \mathbf{x}} = -\frac{\mathrm{d}\psi(\mathbf{x})}{\mathrm{d}\mathbf{x}} \end{aligned} \right\} \Rightarrow (\mathbf{x}',\mathbf{p}')$$

gradients of the pdf

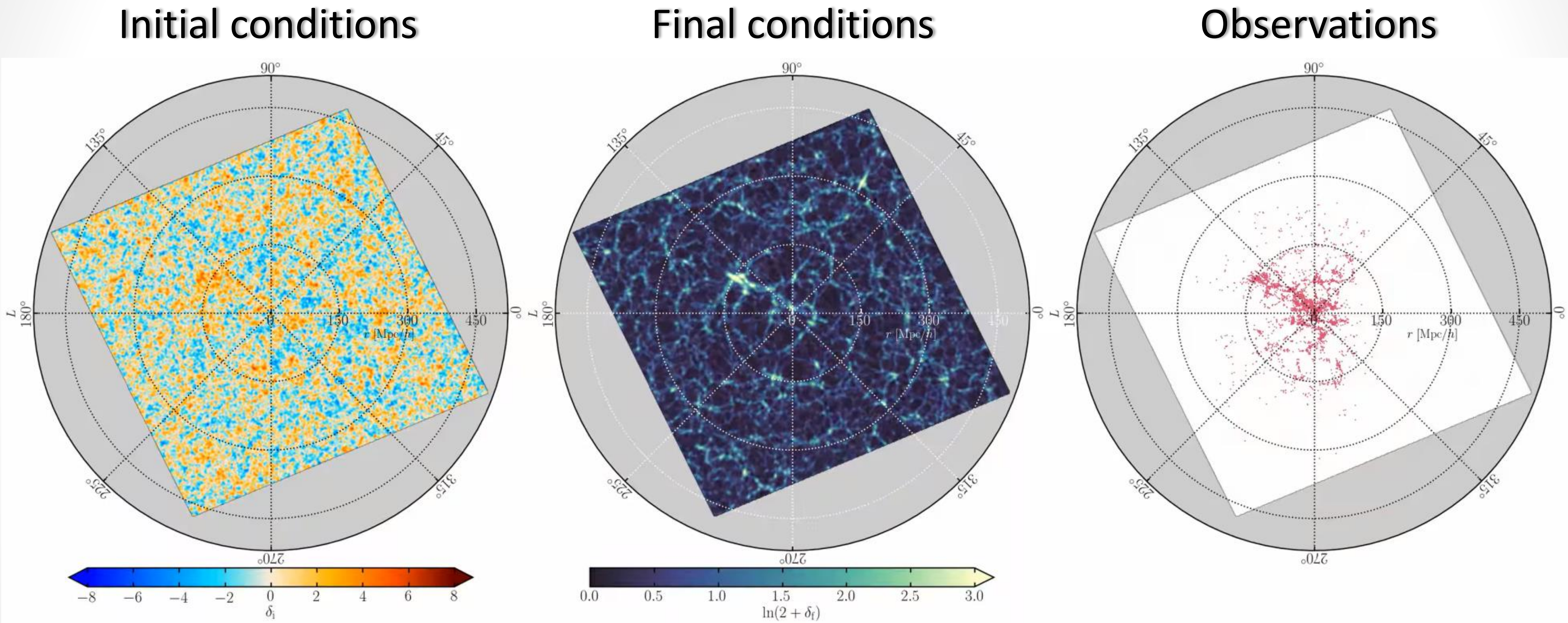
$$a(\mathbf{x}',\mathbf{x}) = \mathrm{e}^{-(H'-H)} = 1$$

acceptance ratio unity

- HMC beats the curse of dimensionality by:
  - Exploiting gradients
  - Using conservation of the Hamiltonian

Duane et al. 1987, Phys. Lett. B 195, 2

BORG at work: Bayesian chrono-cosmography

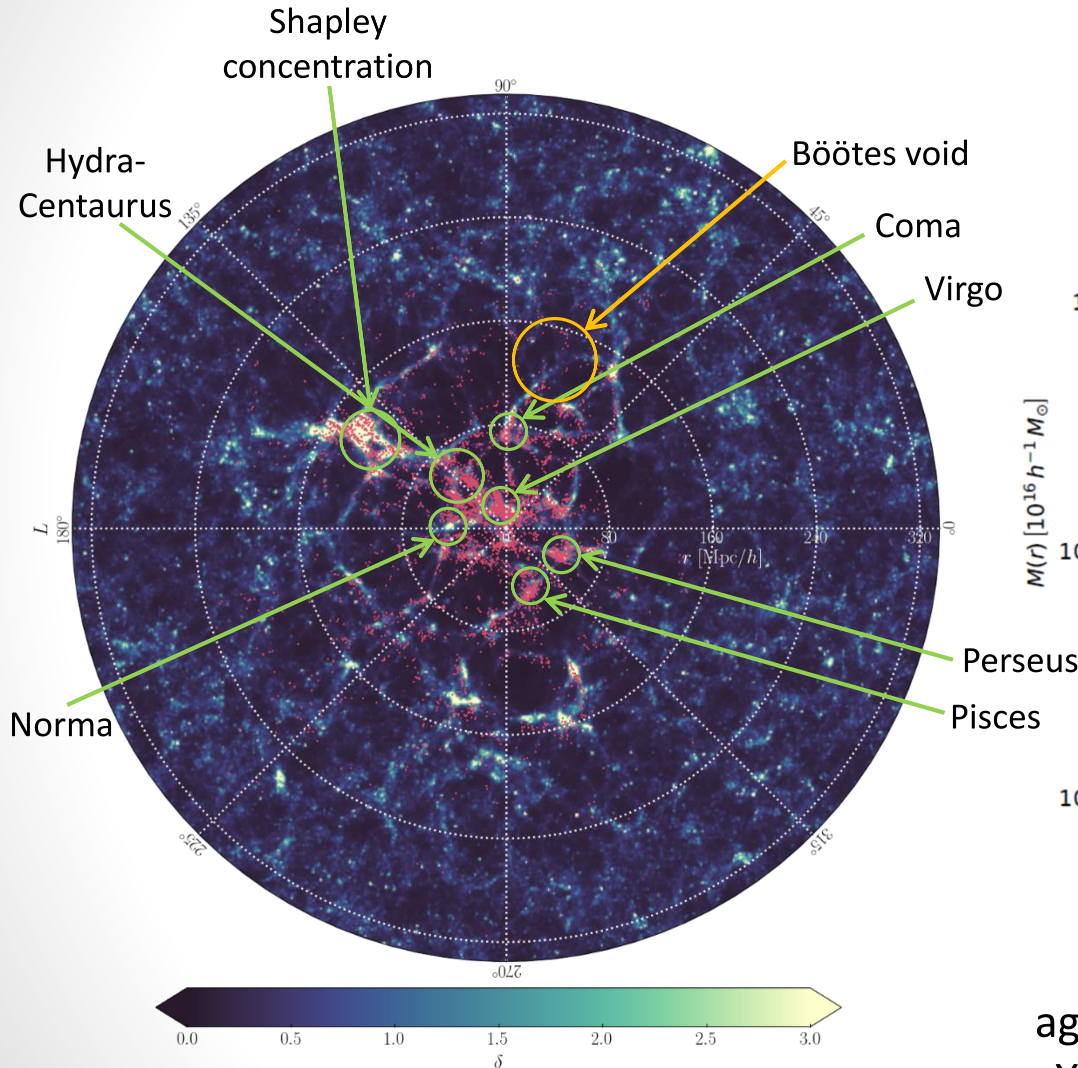


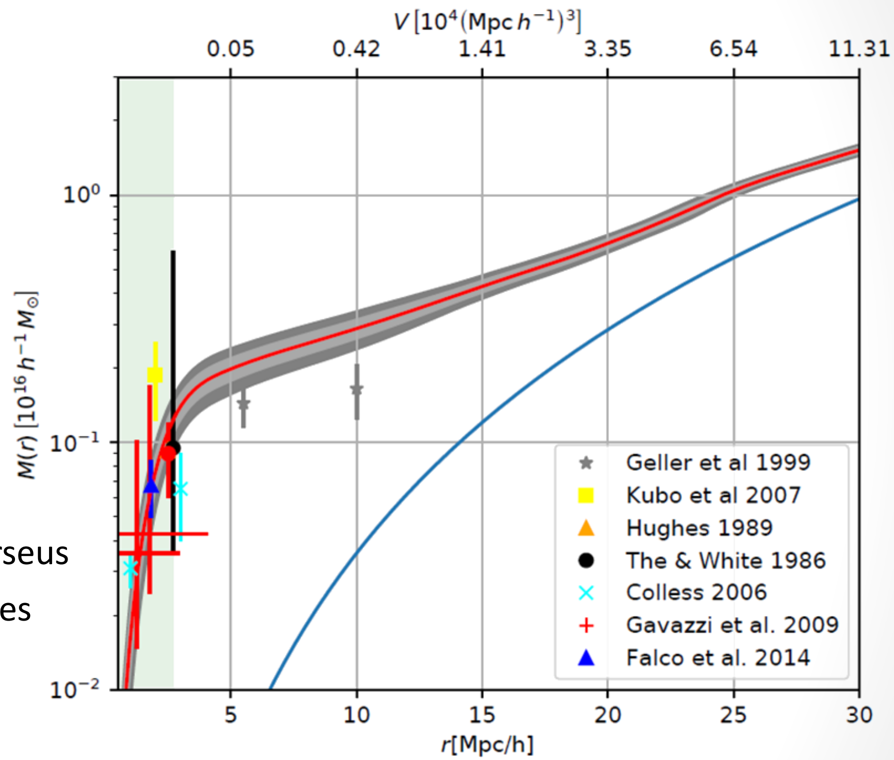
Supergalactic plane

67,224 galaxies, ≈ 17 million parameters, 5 TB of primary data products, 10,000 samples, ≈ 500,000 forward and adjoint gradient data model evaluations, 1.5 million CPU-hours

Jasche & Lavaux 2019, 1806.11117 – FL, Lavaux & Jasche, in prep.

BORGPM density field: full non-linear dynamics



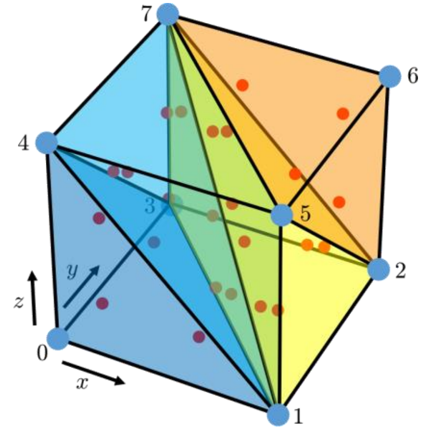


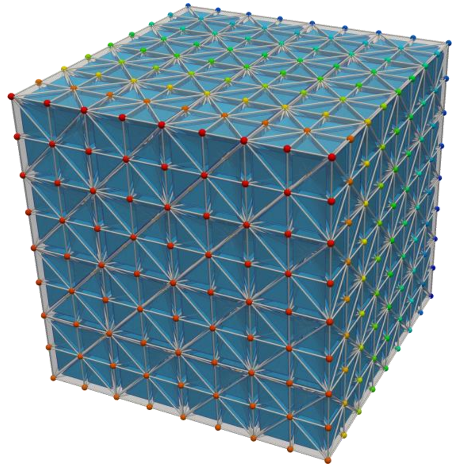
Mass profile of the Coma cluster, in agreement with gravitational lensing and X-ray observations down to a few Mpc.

Jasche & Lavaux 2019, 1806.11117 – FL, Lavaux & Jasche, in prep.

The phase-space structure of dark matter: tools

Delaunay tessellation of elementary Lagrangian cubes



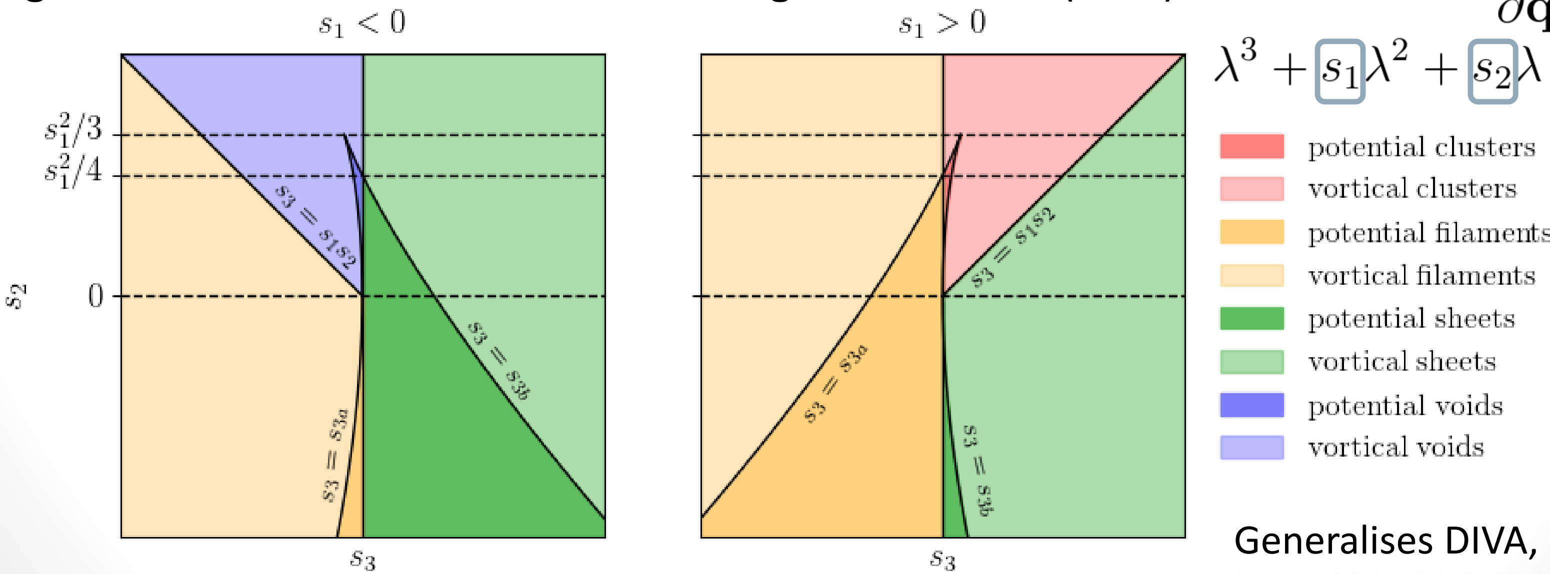


Abel, Hahn & Kaehler 2012, 1111.3944
Shandarin, Habib & Heitmann 2012, 1111.2366
Hahn, Abel & Kaehler 2013, 1210.6652
Hahn & Angulo 2016, 1501.01959
Sousbie & Colombi 2016, 1509.07720

Lagrangian Invariants Classification of Heterogeneous flows (LICH)

$$\mathcal{R}_{\ell m} \equiv \frac{\partial \Psi_\ell}{\partial \mathbf{q}_m}$$

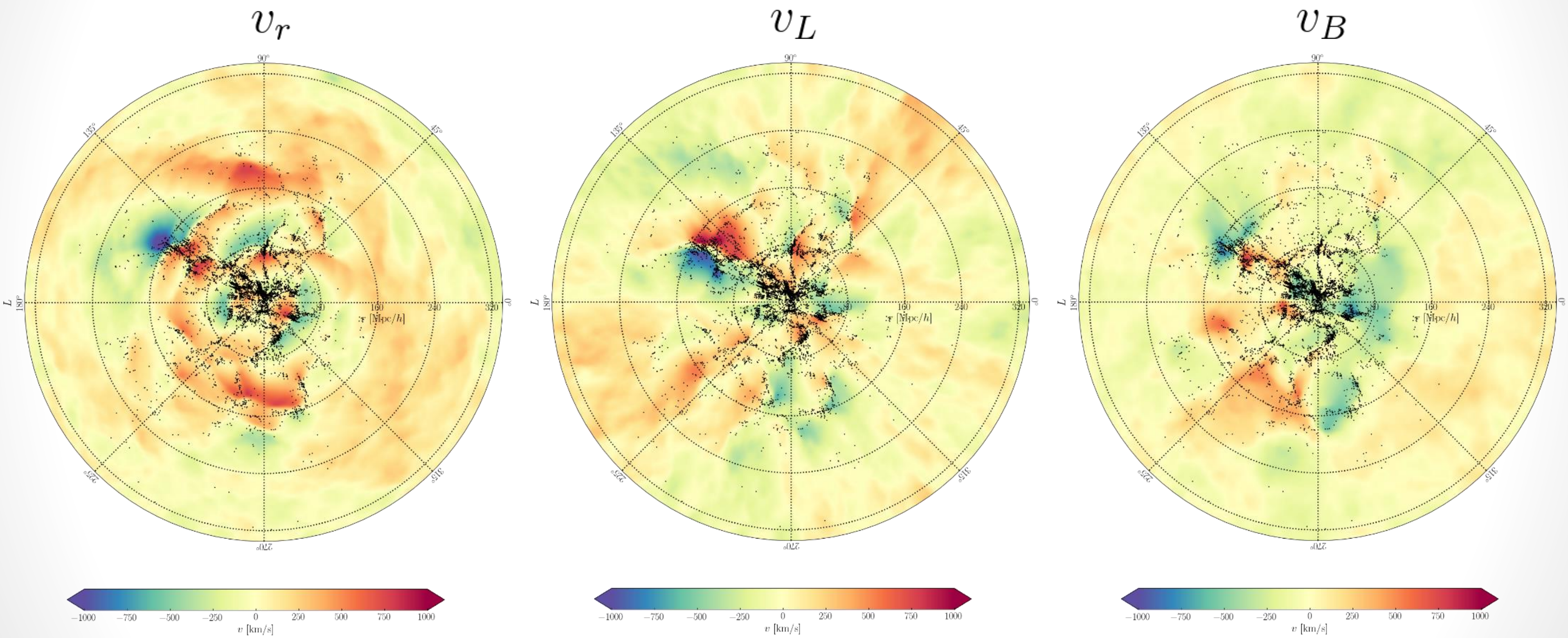
$$\lambda^3 + s_1 \lambda^2 + s_2 \lambda + s_3 = 0$$



FL, Jasche, Lavaux, Wandelt & Percival 2017, 1601.00093

Generalises DIVA,
Lavaux & Wandelt 2010, 0906.4101

Velocity field in the supergalactic plane

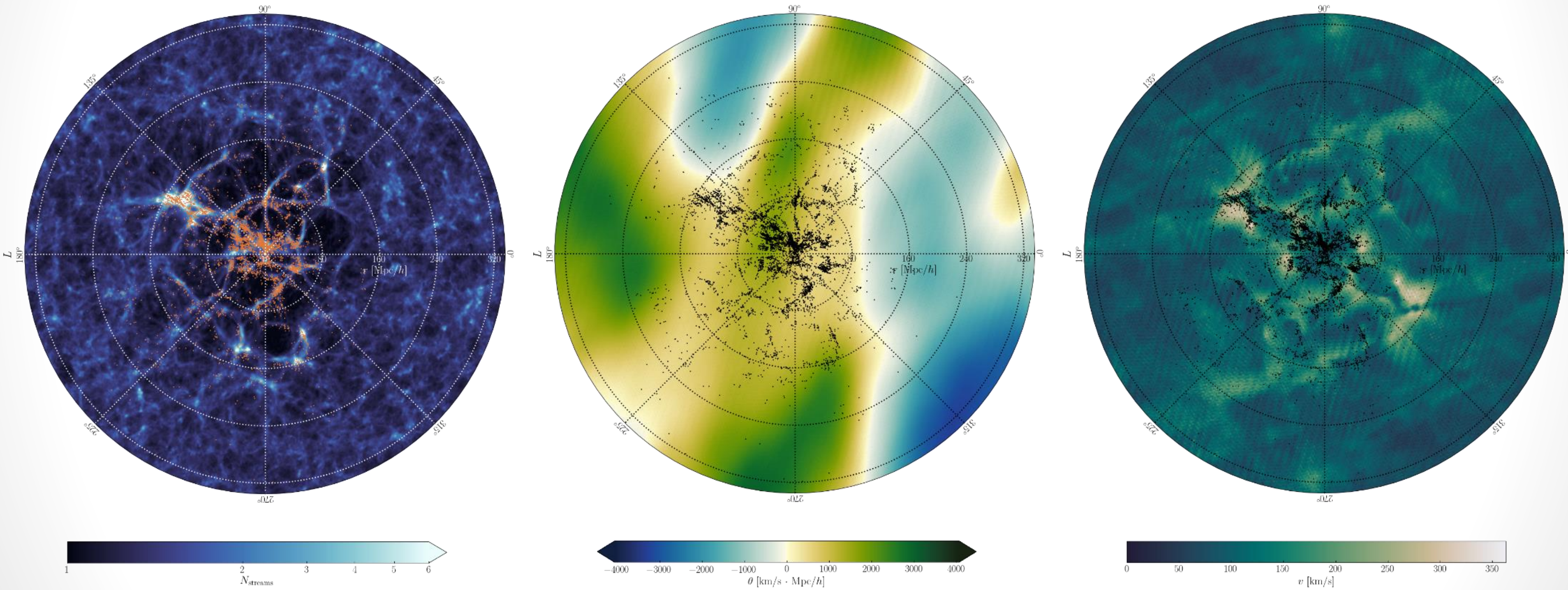


The **gravitational infall** of known structures can be observed.

FL, Lavaux & Jasche, in prep.

Number of streams and vorticity

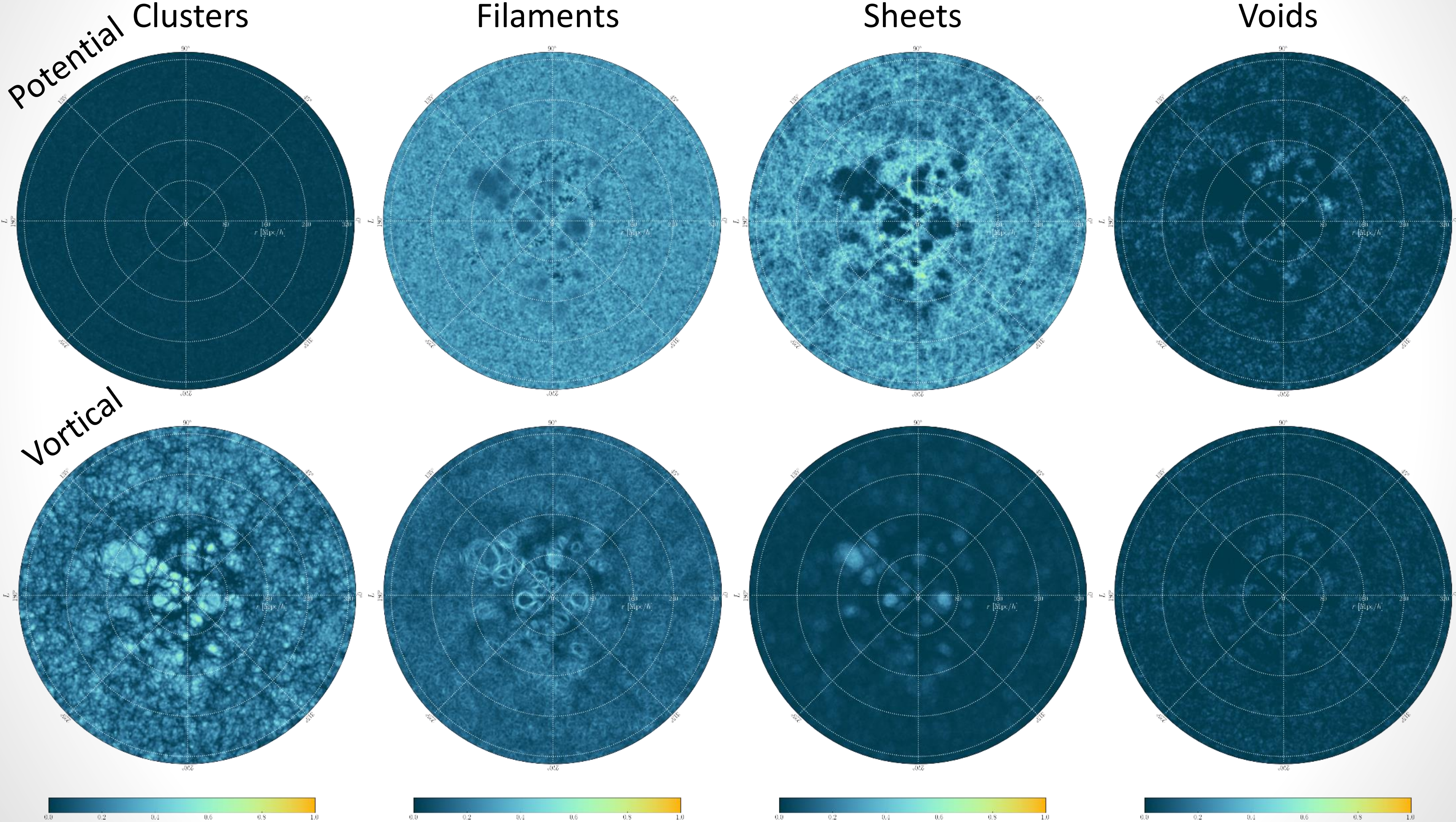
Number of streams Velocity potential Norm of vorticity



In earlier work (Leclercq, Jasche, Lavaux, Wandelt & Percival 2017, 1601.00093), these were postdictions. Thanks to BORGPM (full non-linear dynamics), we have now actual measurements - with uncertainties.

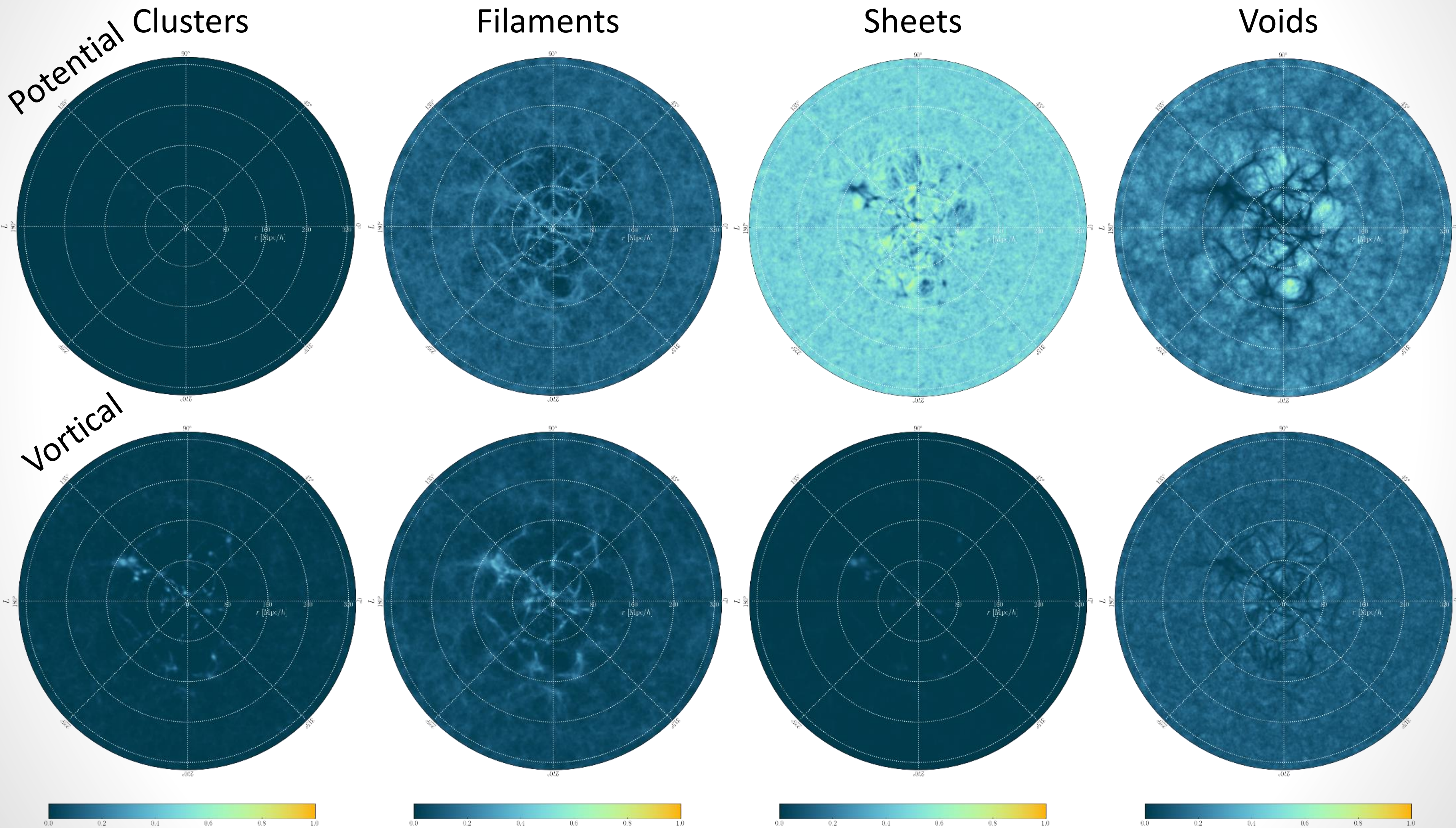
FL, Lavaux & Jasche, in prep.

LICH initial structures inferred by BORG



FL, Lavaux & Jasche, in prep.

LICH final structures inferred by BORG



FL, Lavaux & Jasche, in prep.

Mapping the Universe: epilogue?

J. Cham – PhD comics

Likelihood-free solution: SELFI

Simulator Expansion for Likelihood-Free Inference

Likelihood-based solution:
Exact statistical analysis
Approximate data model

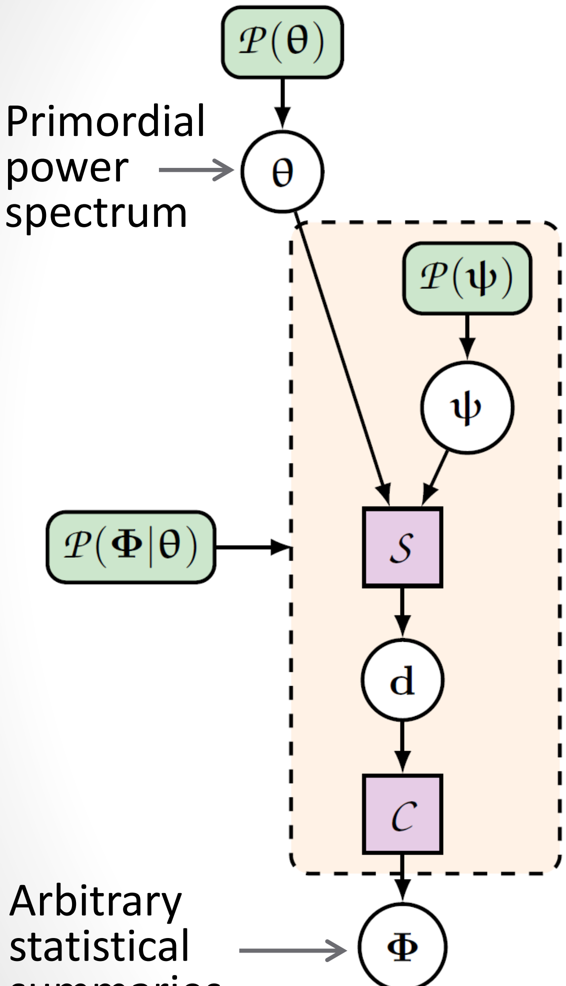
Data assimilation

?

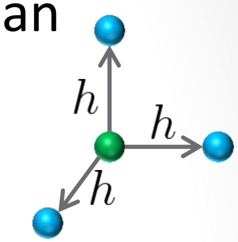
Likelihood-free solution:
Approximate statistical analysis
Arbitrary data model

Generative inference

SELFI: Method



- Gaussian prior + Gaussian effective likelihood
- Linearisation of the black-box around an expansion point + finite differences:



$$\hat{\Phi}_\theta \approx \mathbf{f}_0 + \nabla \mathbf{f}_0 \cdot (\theta - \theta_0)$$

➡ The posterior is Gaussian and analogous to a Wiener filter:

expansion point → θ_0; observed summaries → Φ_O

$$\gamma \equiv \theta_0 + \Gamma\,(\nabla \mathbf{f}_0)^\intercal\, \mathbf{C}_0^{-1}(\Phi_O - \mathbf{f}_0)$$

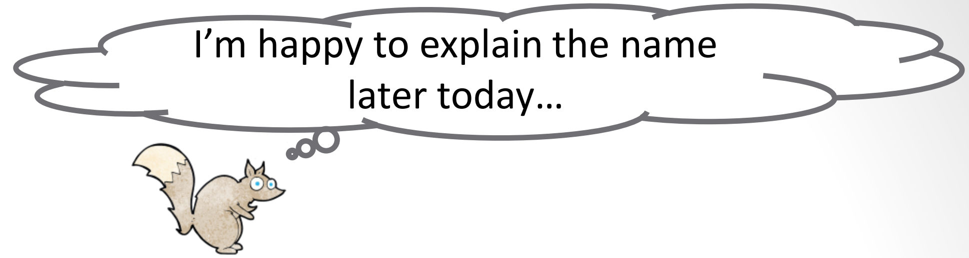
$$\Gamma \equiv \left[(\nabla \mathbf{f}_0)^\intercal\, \mathbf{C}_0^{-1} \nabla \mathbf{f}_0 + \mathbf{S}^{-1}\right]^{-1}$$

covariance of summaries → $\mathbf{C}_0$; gradient of the black-box → $\nabla \mathbf{f}_0$; prior covariance → $\mathbf{S}$

$\mathbf{f}_0, \mathbf{C}_0$ and $\nabla \mathbf{f}_0$ can be evaluated through simulations only.
The number of required simulations is fixed *a priori*.

FL, Enzi, Jasche & Heavens 2019, 1902.10149

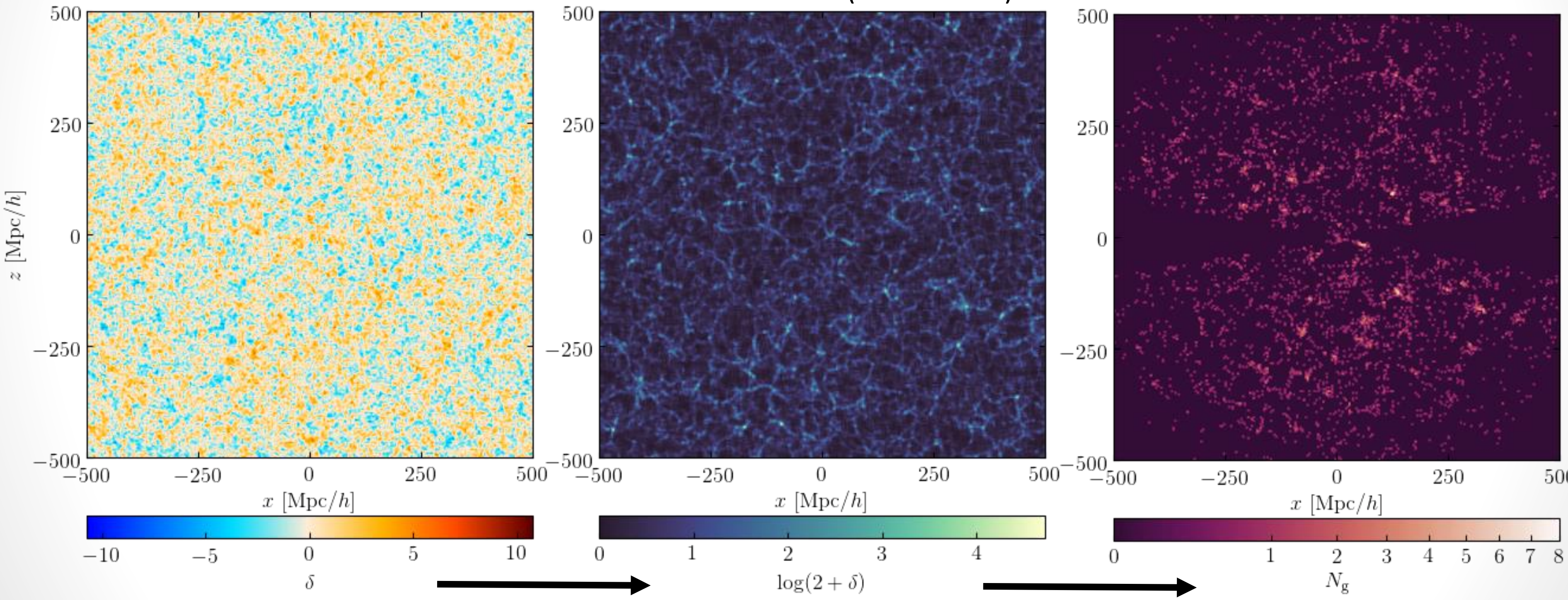
A black-box: Simbelmynë



Publicly available code:

https://bitbucket.org/florent-leclercq/simbelmyne/

Initial conditions | Final conditions (dark matter) | Galaxies

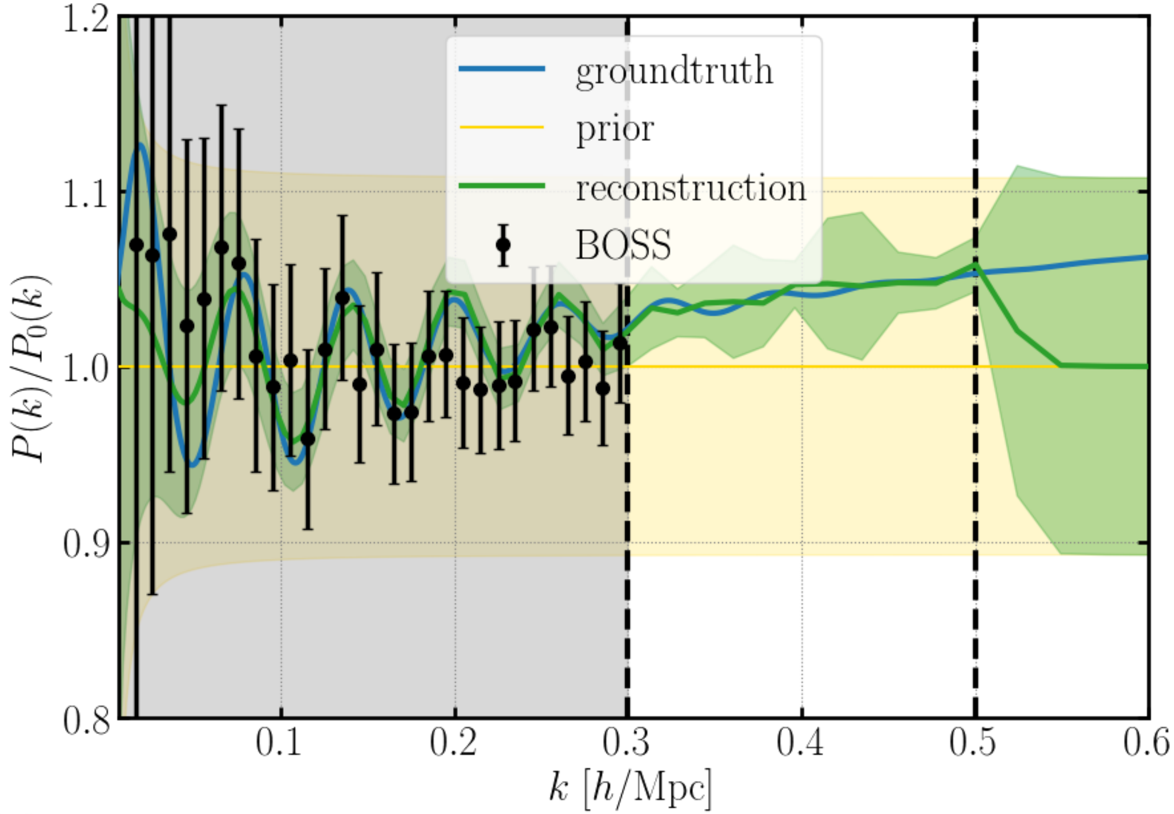


Dark matter simulation with COLA

Tassev, Zaldarriaga & Eisenstein 2013, 1301.0322

Survey simulation:
Redshift-space distortions, galaxy bias, selection effects, survey geometry, instrumental noise

SELFI + Simbelmynë: Proof-of-concept



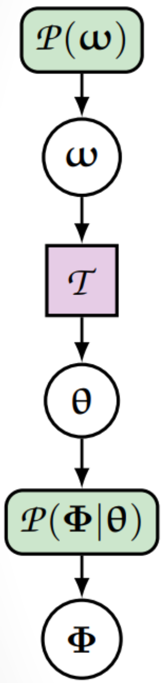
100 parameters are simultaneously inferred from a black-box data model

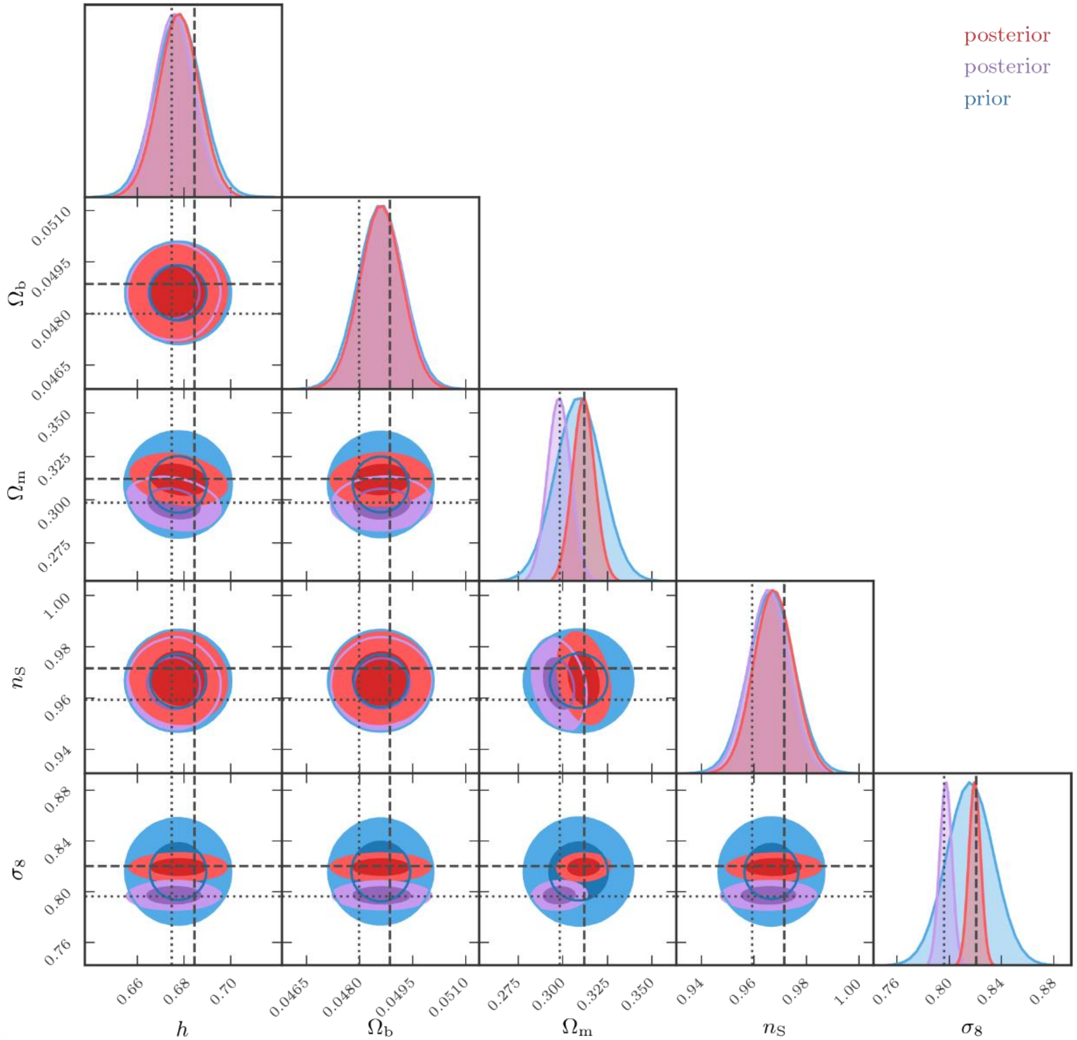
$N_{\mathrm{modes}} \propto k^3$: **5** times more modes are used in the analysis

1 (Gpc/h)^3 only! Much more potential for Euclid data…

FL, Enzi, Jasche & Heavens 2019, 1902.10149

SELFI + Simbelmynë: Proof-of-concept

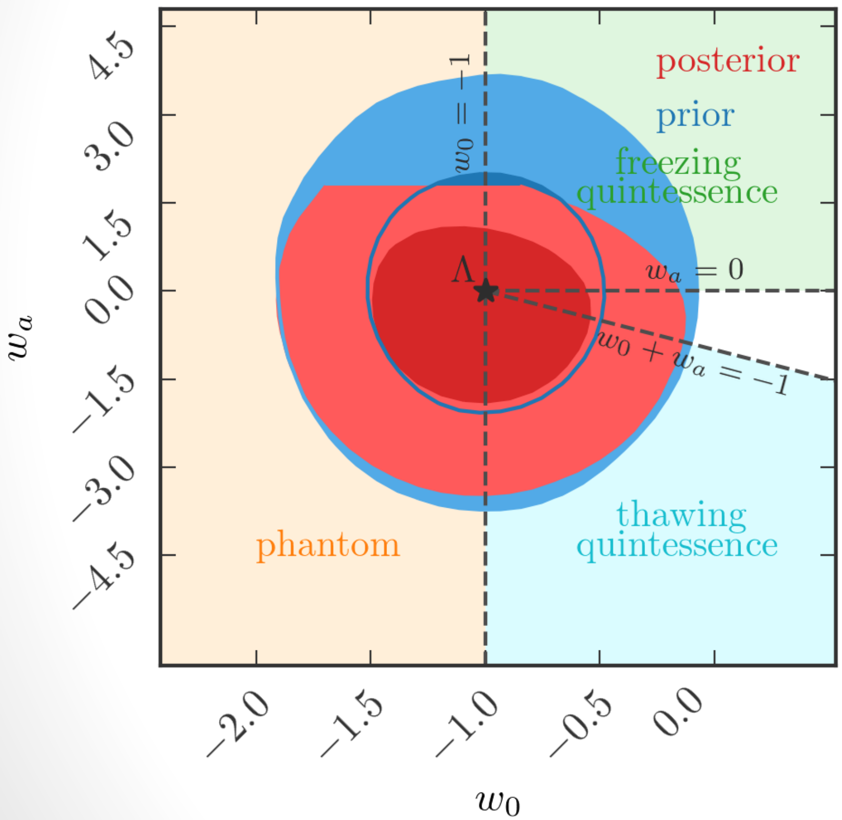


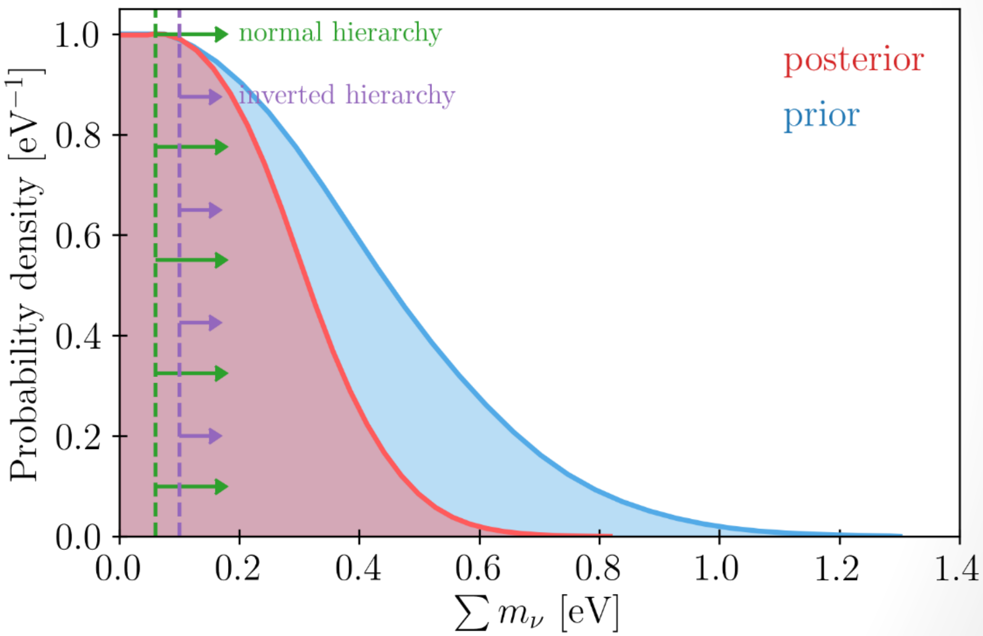


- Robust inference of cosmological parameters can be easily performed *a posteriori* once the linearised data model is learnt

FL, Enzi, Jasche & Heavens 2019, 1902.10149

Dark energy and neutrino masses with SELFI



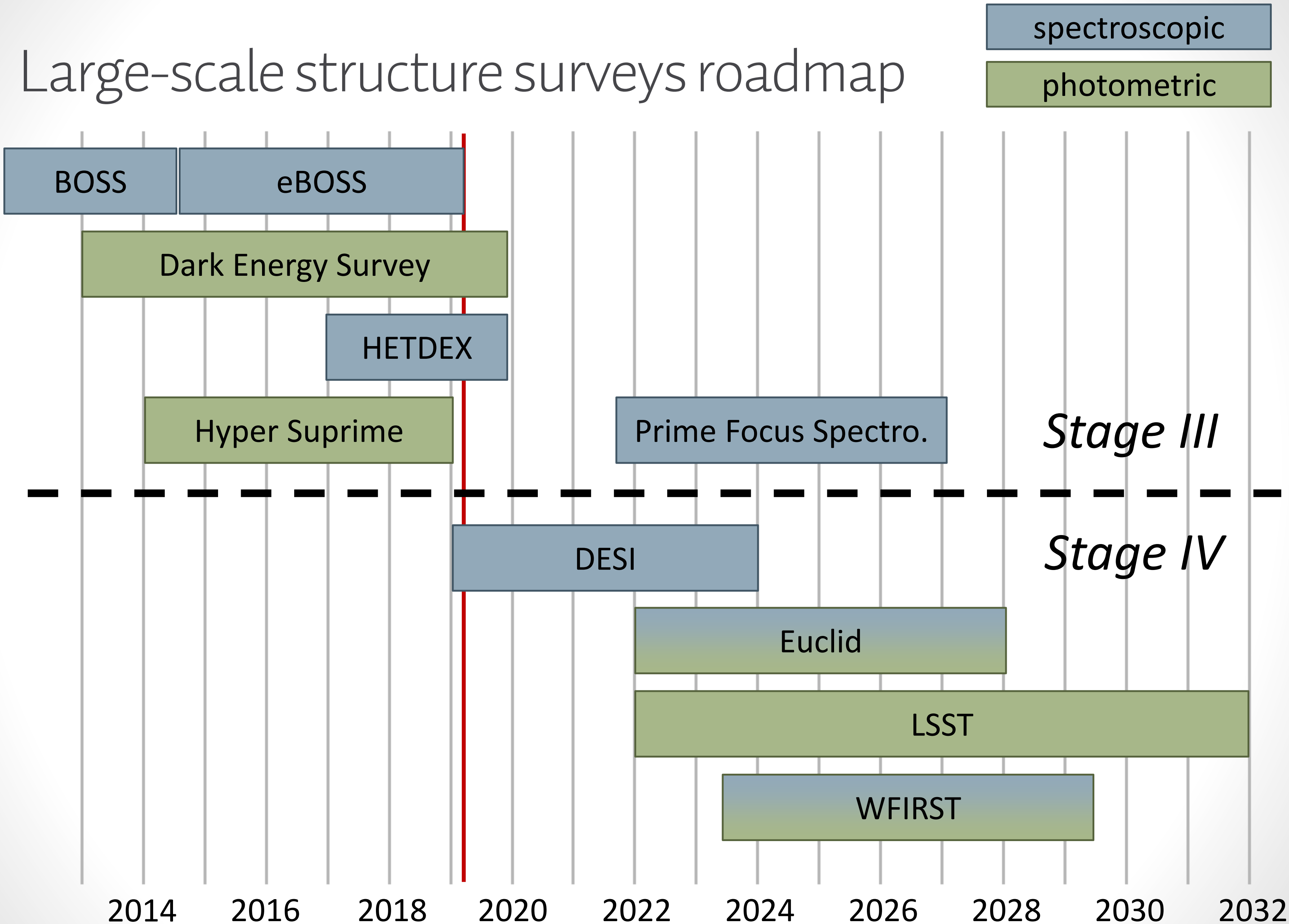


pyselfi will be made publicly available soon.

The Future: Opportunities & Challenges

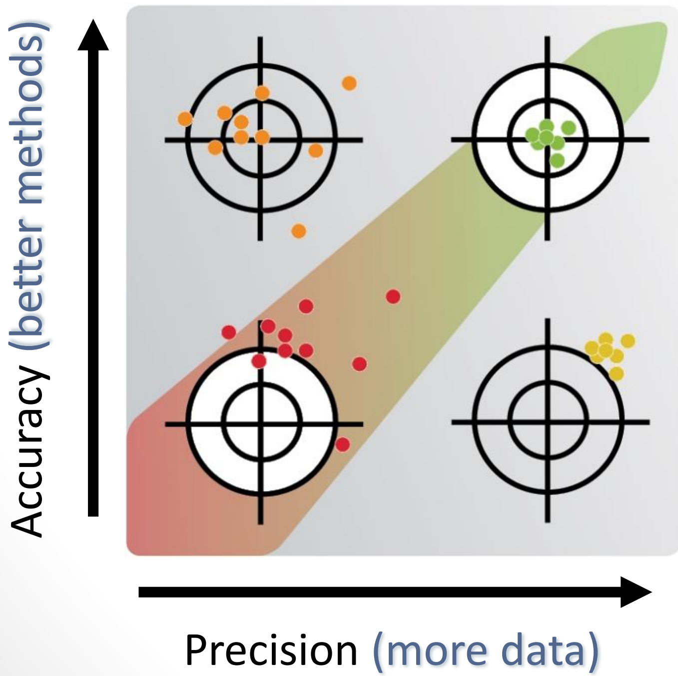
DESI, Euclid, LSST, WFIRST, and more…

Large-scale structure surveys roadmap



Data-intensive scientific discovery from galaxy surveys

- Next-generation surveys will be dominated by systematics
- 80% of the total signal will come from non-linear structures

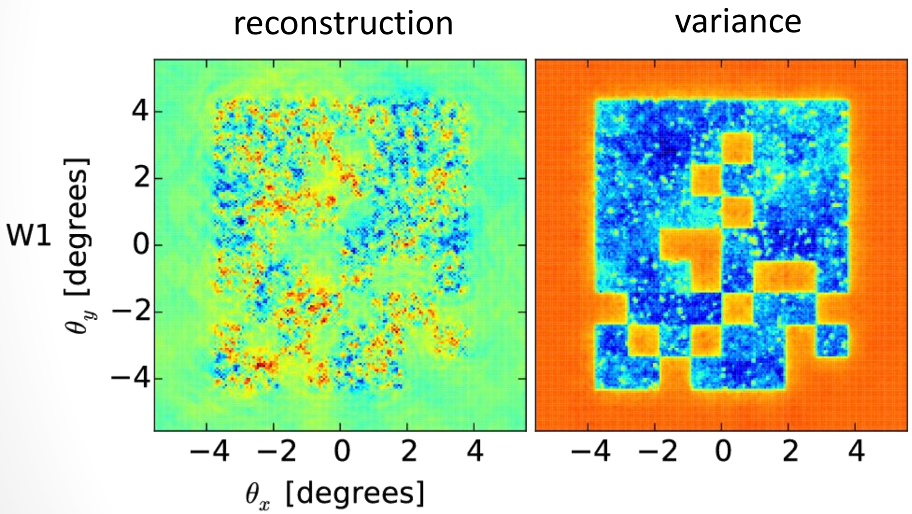


- Can data analysts keep pace?

The Imperial weak lensing inference framework

with George Kyriacou, Arrykrishna Mootoovaloo, Alan Heavens & Andrew Jaffe

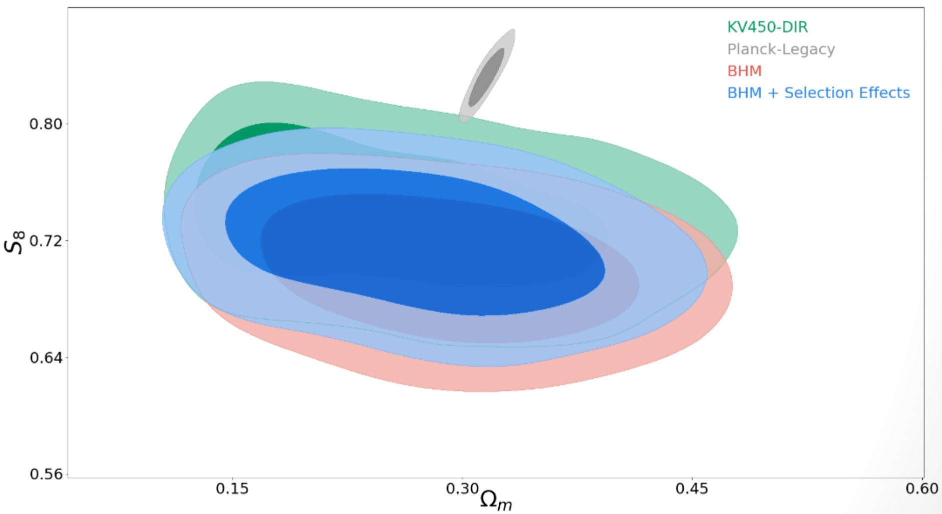
Joint inference of cosmic shear maps and power spectra/cosmology from CFHTLenS



$$\sum m_\nu < 4.6\ \mathrm{eV}(95\%)$$ from lensing data alone

Alsing, Heavens & Jaffe 2016, 1607.00008

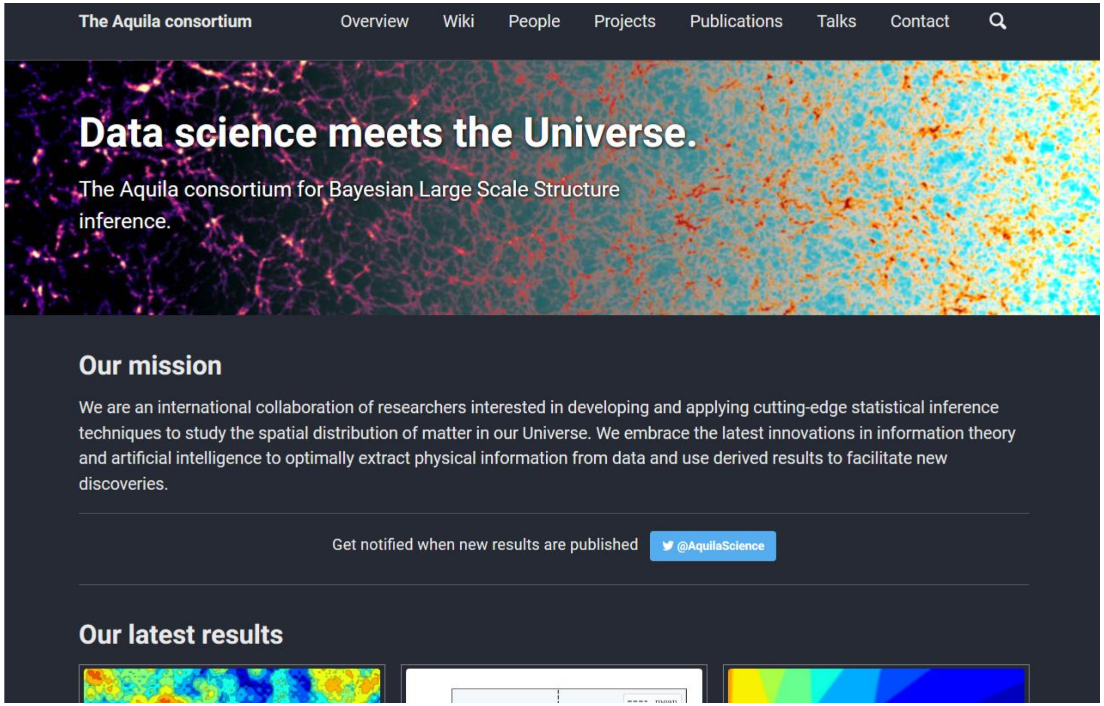
Bayesian hierarchical inference of galaxy redshift distributions $n(z)$



Kyriacou, FL, Heavens & Jaffe, in prep.

The Aquila Consortium

- Created in 2016. Members from the UK, France, Germany & Sweden.
- Gathers people interested in developing the Bayesian pipelines and running analyses on cosmological data.



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Concluding thoughts

Likelihood-based solution:
Exact statistical analysis
Approximate data model

Data assimilation

?

Likelihood-free solution:
Approximate statistical analysis
Arbitrary data model

Generative inference

- Bayesian analyses of galaxy surveys with fully non-linear numerical models is not an impossible task!
- A likelihood-based solution (BORG): general purpose reconstruction of dark matter from galaxy clustering, providing new measurements and predictions
- A likelihood-free solution (SELFI): algorithm for targeted questions, allowing the use of simulators including all relevant physical and observational effects

Concluding thoughts

- The future: great science and challenges

Galaxy formation: bias model & likelihood
Large volume, photometric redshifts
Instruments modelling

DESI, Euclid, LSST, WFIRST

Bayesian large-scale structure analyses

Cosmological measurements:

- Cosmic expansion
- Power spectrum (and governing parameters)
- Gaussianity tests of the initial conditions
- Dark energy from the growth of structure

Predictive cosmology:

- Velocity field
- X-ray cluster emission
- Gravitational lensing
- CMB secondary effects
- Dark matter?