Optimizing Lagrangian perturbation theory in the mildly nonlinear regime of cosmic structure formation via one-point remapping

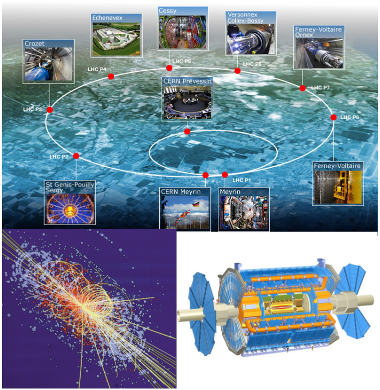
Florent Leclercq

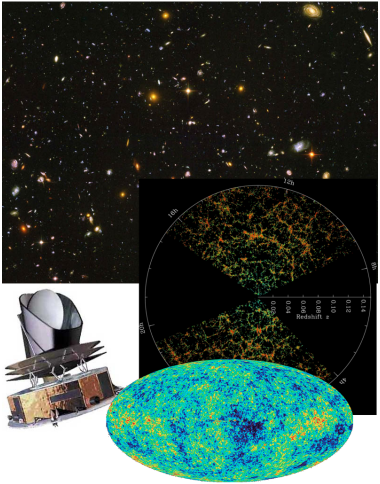
Institut d'Astrophysique de Paris

April 29th, 2013

In collaboration with: Jens Jasche (IAP), Héctor Gil-Marín (U. Portsmouth/U. Barcelona), Benjamin D. Wandelt (IAP/U. Illinois)

Showdown: Particle accelerators vs cosmological observations





The inhomogeneous Universe

You are here, make the best of it...

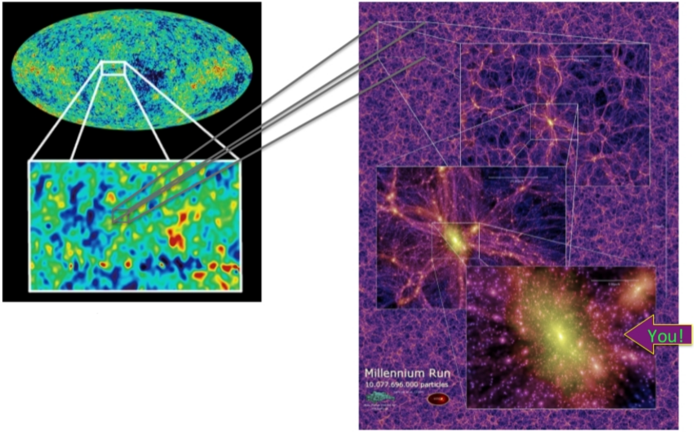


Figure: Left: Primordial perturbations as seen in the Cosmic Microwave Background anisotropies (WMAP)
Right: Dark matter distribution today (simulated)

Issues and methods in cosmostatistics

Cosmostatistics: discipline of **using the departures from homogeneity** observed in astronomical surveys to **distinguish between cosmological models**.

Huge data sets, but fundamental limits to information:

- on large scales: causality
- on small scales: non-linearity

Large scales: careful statistical treatment required (cosmic variance).

Intermediate scales: linear methods are suitable.

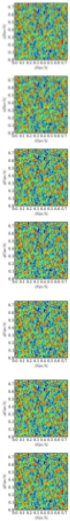
Small scales: number of accessible modes in a 3D galaxy survey $\propto k^3$
$\Rightarrow$ LSS surveys allow probing a larger number of small-scale modes in the *midly non-linear* regime (the 3D "cosmological revolution").

Bayesian cosmostatistics

- No ideal observation in reality: statistical and systematic uncertainties, noise, cosmic variance, survey geometry, selection effects, biases, etc.
  $\Rightarrow$ no unique recovery of the initial conditions is possible!
- $\Rightarrow$ a good question: "What is the probability distribution of possible signals compatible with the observations?"

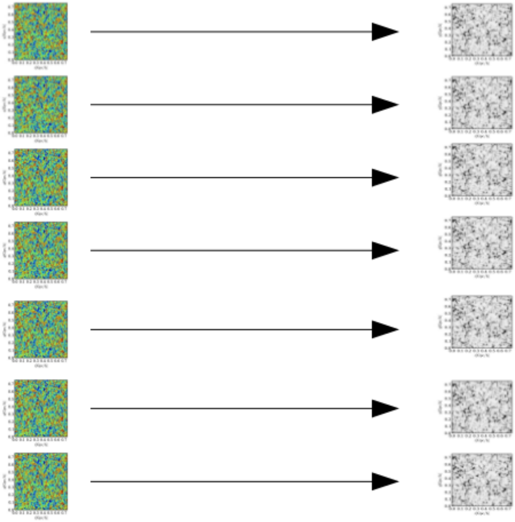
4D physical inference of the initial conditions

Jasche & Wandelt 2012



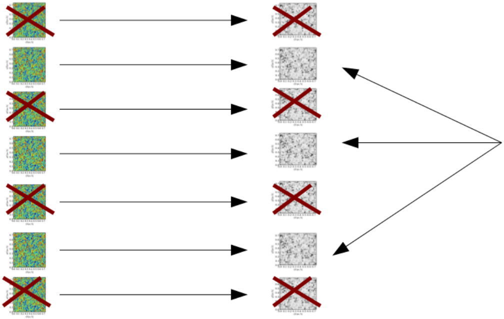
4D physical inference of the initial conditions

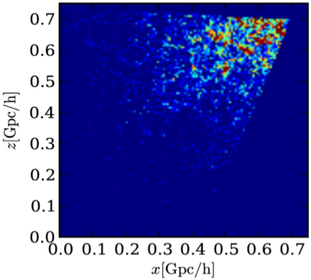
Jasche & Wandelt 2012



4D physical inference of the initial conditions

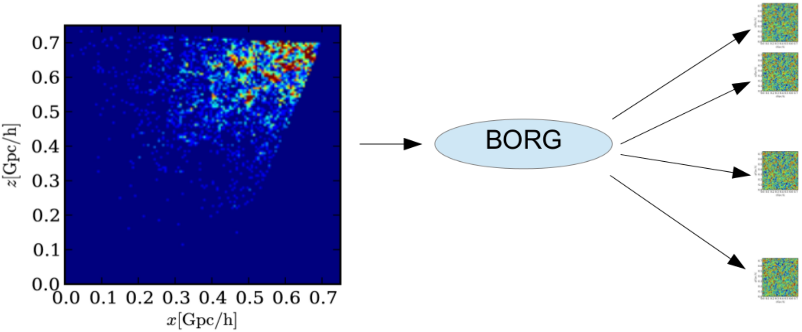
Jasche & Wandelt 2012





4D physical inference of the initial conditions

Jasche & Wandelt 2012



BORG (Bayesian Origin Reconstruction from Galaxies):

- Hamiltonian Monte-Carlo
- 2LPT

The Vlasov-Poisson system

Standard picture of LSS formation: result of gravitational amplification of primordial fluctuations of the initial density field.

1. gravitational aggregation of cold dark matter (CDM) particles
2. condensation of baryonic matter in gravitational potentials wells formed by the dark matter distribution

Modelization of the first step: in terms of Newtonian dynamics in comoving coordinates and conformal time (we follow the Hubble expansion flow and are interested in fluctuations rather than mean quantities).

The Vlasov-Poisson system

$$\frac{\mathrm{d}f}{\mathrm{d}\tau} = \frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{ma} \cdot \nabla f - ma\nabla\Phi \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$

$$\Delta\Phi = 4\pi \mathrm{G} a^2 \bar{\rho}\delta$$

Φ: cosmological gravitational potential, δ: density contrast, f: particle number density in phase space, $\mathbf{p} = ma\mathbf{u}$: momentum for *peculiar* velocity

Fluid dynamics approach

The Vlasov-Poisson system is *non-linear*. A common approach is to take momentum moments of the Vlasov equation → hierarchy of equations, truncated at some point with a fluid dynamics assumption.

Zeroth moment: conservation of mass

Continuity equation

$$\frac{\partial \delta(\mathbf{x},\tau)}{\partial \tau} + \nabla \cdot \{[1+\delta(\mathbf{x},\tau)]\,\mathbf{u}(\mathbf{x},\tau)\} = 0$$

First moment: conservation of momentum

Euler equation

$$\frac{\partial \mathbf{u}_i(\mathbf{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\mathbf{u}_i(\mathbf{x},\tau) + \mathbf{u}_j(\mathbf{x},\tau)\cdot\nabla_j \mathbf{u}_i(\mathbf{x},\tau) = -\nabla_i \Phi(\mathbf{x},\tau) - \frac{1}{\rho(\mathbf{x},\tau)}\nabla_j(\sigma_{ij}(\mathbf{x},\tau))$$

The single-stream approximation

At early stages of cosmological gravitational instability or at large scales,

- structures had no time to collapse,
- gravity-induced cosmic flows will dominate over velocity dispersions due to thermal motion.

The *single-stream approximation*:

- the stress tensor is negligible: $\sigma_{ij} \approx 0$,
- density in phase space satisfies $f(\mathbf{x}, \mathbf{p}, \tau) = \rho(\mathbf{x}, \tau)\, \delta_\mathrm{D}[\mathbf{p} - ma\mathbf{u}(\mathbf{x})]$.

Breakdown of the approximation: *shell-crossing*:

- generation of velocity dispersion and anisotropic pressure,
- multiple streams at a single point.

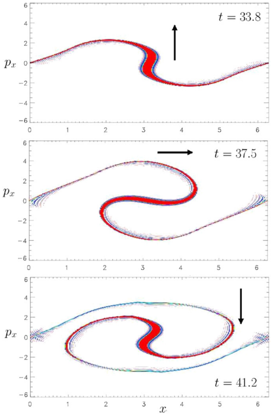


Figure: Inglebert *et al.* Plasma Phys. Control. Fusion **54** (2012)

The Zel'dovich approximation (ZA)

- As in fluid mechanics, there are two ways to describe the cosmological fluid: Eulerian and Lagrangian. We focus on the Lagrangian approach:

$$\mathbf{x}(\tau) = \mathbf{q} + \Psi(\mathbf{q}, \tau)$$

$\mathbf{q}$: initial position, $\mathbf{x}$: final position, Ψ: displacement field

- The Zel'dovich approximation (ZA) = first order Lagrangian perturbation theory. The linear solution to the dynamics is:

$$\psi^{(1)}(\mathbf{q}, \tau) = \nabla_{\mathbf{q}} \cdot \Psi^{(1)}(\mathbf{q}, \tau) = -D_1(\tau)\delta(\mathbf{q})$$

$D_1(\tau)$: linear growth factor

  - In comoving coordinates particles just go straight in the direction set by their initial velocity.
  - *Local* approximation: does not depend on the behavior of the rest of fluid elements.

Second-order Lagrangian perturbation theory (2LPT)

- The ZA fails at sufficiently non-linear stages when particles are forming gravitationally bound structures instead of following straight lines. 2LPT provides a remarkable improvement over the ZA in describing the global properties of density and velocity fields.
- 2LPT is *non-local*, i.e. it includes the correction to the ZA displacement due to gravitational tidal effects.

$$\mathbf{x}(\tau) = \mathbf{q} + \Psi^{(1)}(\mathbf{q},\tau) + \Psi^{(2)}(\mathbf{q},\tau)$$

$$\psi^{(1)}(\mathbf{q},\tau) = \nabla_{\mathbf{q}} \cdot \Psi^{(1)}(\mathbf{q},\tau) = -D_1(\tau)\delta(\mathbf{q})$$

$$\psi^{(2)}(\mathbf{q},\tau) = \nabla_{\mathbf{q}} \cdot \Psi^{(2)}(\mathbf{q},\tau) = \frac{1}{2}\frac{D_2(\tau)}{D_1^2(\tau)} \sum_{i \neq j} \left[\Psi^{(1)}_{i,i}\Psi^{(1)}_{j,j} - \Psi^{(1)}_{i,j}\Psi^{(1)}_{j,i} \right]$$

with the *second-order growth factor*:

$$D_2(\tau) \approx -\frac{3}{7} D_1^2(\tau)$$

The Eulerian remapping procedure

Due to mode coupling, positive and negative fluctuations grow at different rates in the non-linear regime, but even non-linear evolution tends to preserve the *rank order* of the pixels, sorted by density.

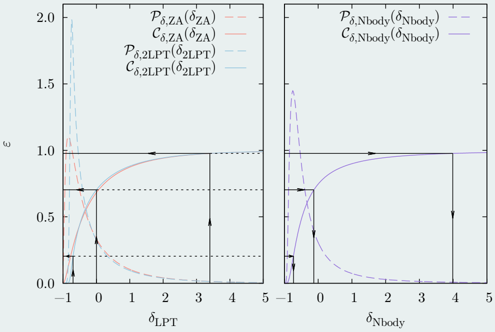
$\Rightarrow$ Remapping algorithm:

- keep positions of under- and over-densities predicted by LPT
- at pixel of rank order δ_{LPT}, assign a new density δ_{Nbody}

$\mathcal{P}_{\mathrm{LPT}}$, $\mathcal{P}_{\mathrm{Nbody}}$: PDFs for the density contrast.

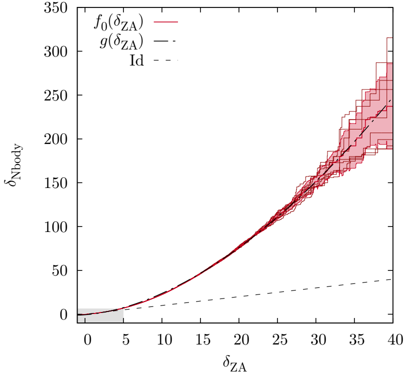
$\mathcal{C}_{\mathrm{LPT}}$, $\mathcal{C}_{\mathrm{Nbody}}$: the corresponding CDFs (their integrals).

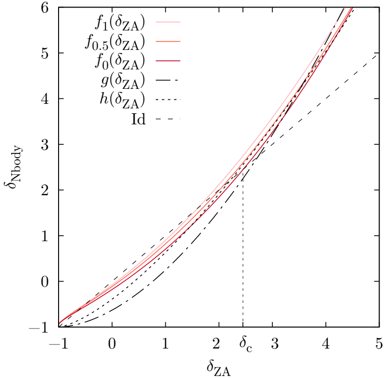
$$\mathcal{C}_{\mathrm{LPT}}(\delta_{\mathrm{LPT}}) = \mathcal{C}_{\mathrm{Nbody}}(\delta_{\mathrm{Nbody}})$$



The Eulerian remapping function for the ZA

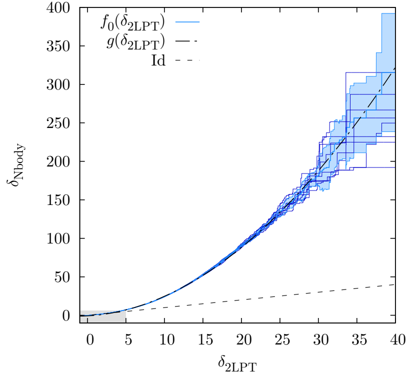
$$f(\delta_{\mathrm{ZA}}) \equiv \delta_{\mathrm{Nbody}} = \mathcal{C}^{-1}_{\mathrm{Nbody}}(\mathcal{C}_{\mathrm{ZA}}(\delta_{\mathrm{ZA}}))$$

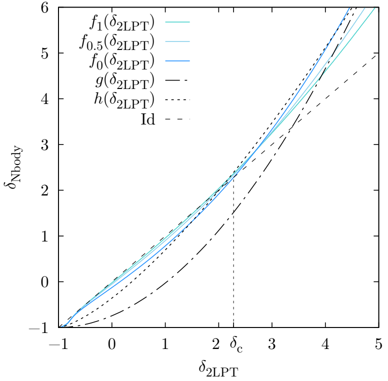




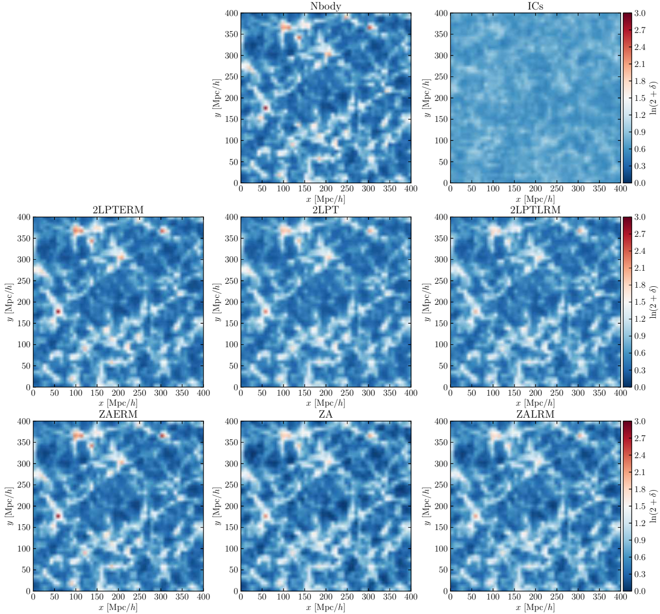
The Eulerian remapping function for 2LPT

$$f(\delta_{\mathrm{2LPT}}) \equiv \delta_{\mathrm{Nbody}} = \mathcal{C}^{-1}_{\mathrm{Nbody}}(\mathcal{C}_{\mathrm{2LPT}}(\delta_{\mathrm{2LPT}}))$$

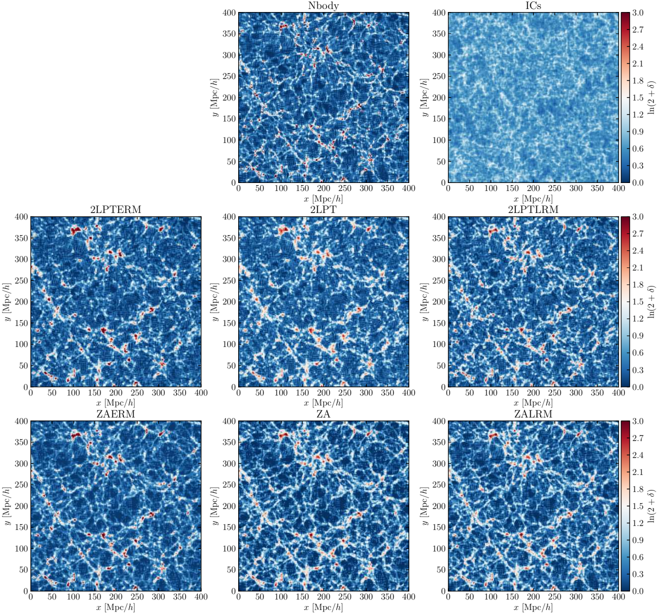




Location of particles



Location of particles



Results: Two-point statistics

How does remapping affect the higher-order correlators?

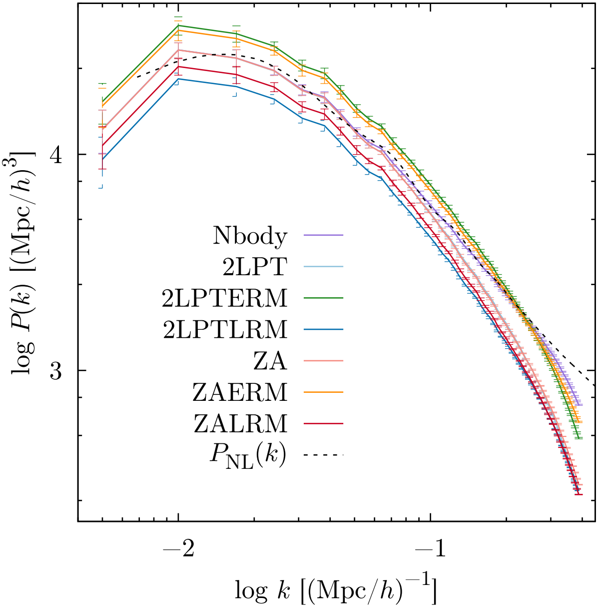
- we expect the higher-order correlations to be respected by the remapping procedure;
- possible improvements could be exploited in data analysis or artificial galaxy survey applications.

Power spectrum:

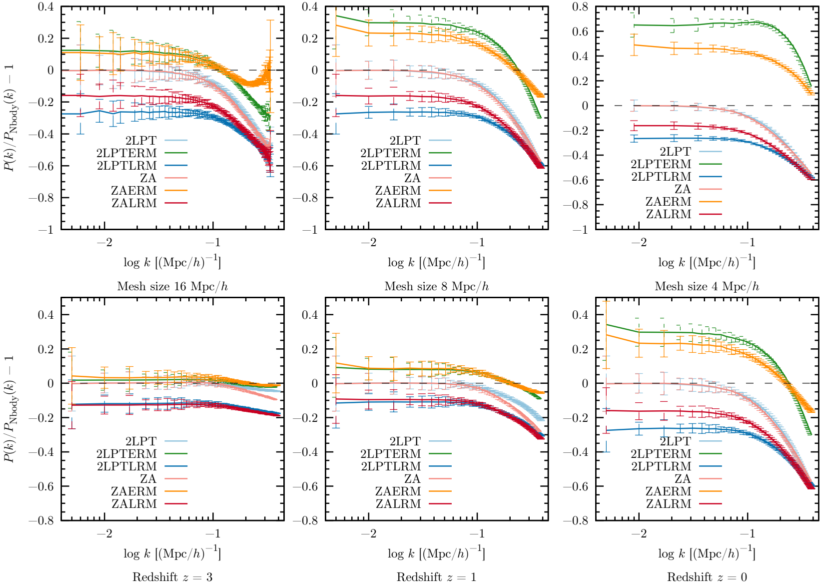
$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle = \delta_{\mathrm{D}}(\mathbf{k}_1 + \mathbf{k}_2)\, P(k)$$

- simplest statistic of interest beyond one-point function
- contains all information for a Gaussian random field (Wick's theorem)
- used in particular to derive the cosmological parameters

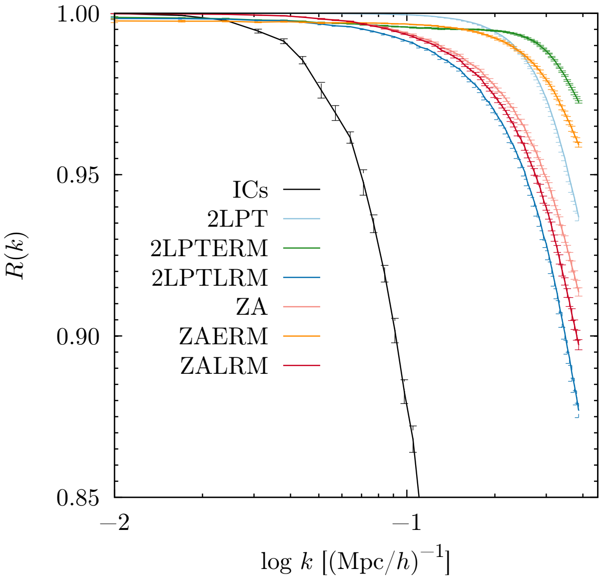
Power spectrum



Power spectrum: varying mesh size and redshift



Fourier-space cross-correlation coefficient



Results: Three-point statistics

Bispectrum:

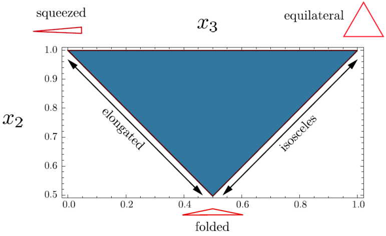
$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = \delta_\mathrm{D}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\, B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

- depends on triangle shape
- provides information on the galaxy bias (simplest model: $\delta_\mathrm{g} = b\,\delta_\mathrm{m}$)
- primordial non-Gaussianity probe

Reduced bispectrum:

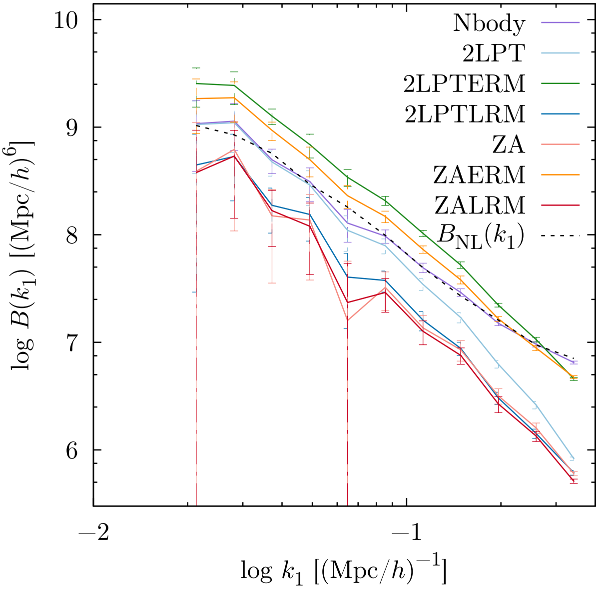
$$Q(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{P(k_1)P(k_2)+P(k_1)P(k_3)+P(k_2)P(k_3)}$$

takes away most of the dependence on scale and cosmology $\Rightarrow$ useful to isolate the effects of gravity (e.g. Gil-Marín *et al.* 2011, JCAP **11**, 019, arXiv:1109.2115)

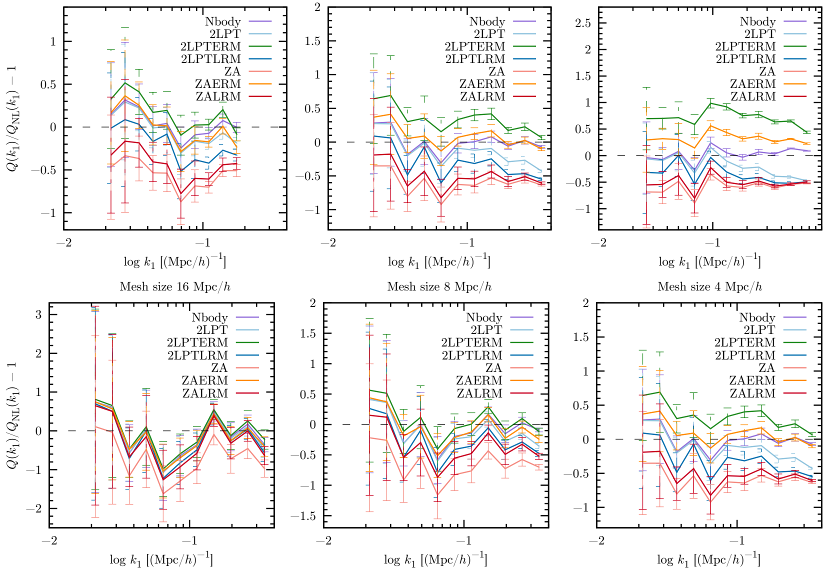


Redshift $z = 0$, mesh size 4 Mpc/h, folded triangles

Bispectrum


Nbody
2LPT
2LPTERM
2LPTLRM
ZA
ZAERM
ZALRM
$B_{\mathrm{NL}}(k_1)$
$\log B(k_1)$ $[(\mathrm{Mpc}/h)^6]$
$\log k_1$ $[(\mathrm{Mpc}/h)^{-1}]$
10
9
8
7
6
−2
−1

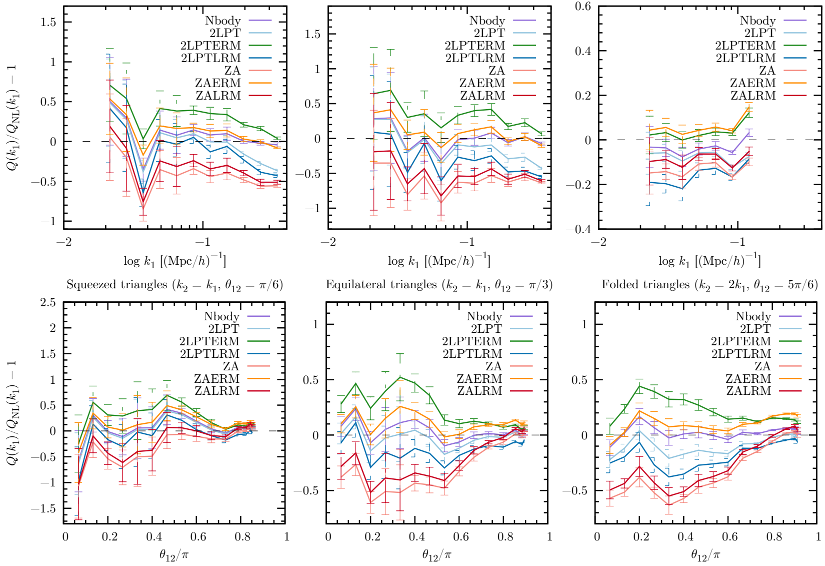
Bispectrum: varying mesh size and redshift



Mesh size 16 Mpc/h — Mesh size 8 Mpc/h — Mesh size 4 Mpc/h

Redshift $z = 3$ — Redshift $z = 1$ — Redshift $z = 0$

Bispectrum: varying triangle shape



Squeezed triangles ($k_2 = k_1$, $\theta_{12} = \pi/6$) — Equilateral triangles ($k_2 = k_1$, $\theta_{12} = \pi/3$) — Folded triangles ($k_2 = 2k_1$, $\theta_{12} = 5\pi/6$)

$k_1 = k_2 = 0.05$ (Mpc/h)$^{-1}$ — $k_1 = k_2 = 0.10$ (Mpc/h)$^{-1}$ — $k_1 = k_2 = 0.15$ (Mpc/h)$^{-1}$

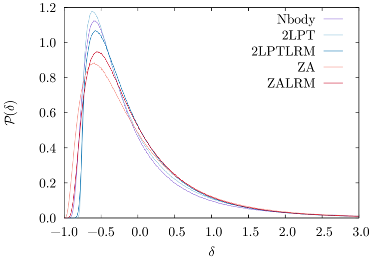
Why the divergence of the displacement field?

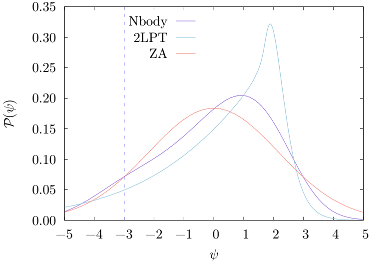
$$\mathbf{x}(\tau) = \mathbf{q} + \Psi(\mathbf{q}, \tau)$$

Reason 1: Ψ is curl-free up to order 2

$\Rightarrow$ Nearly the whole of the information in ψ!

Why the divergence of the displacement field?



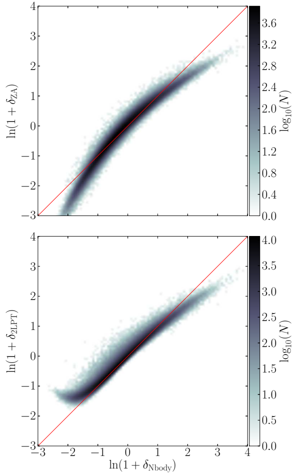


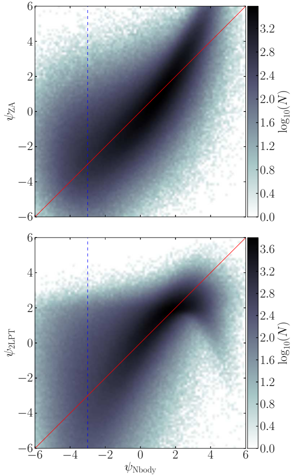
Reason 2: Reduced non-Gaussianity!

Further comments on ψ

see also Neyrinck 2013

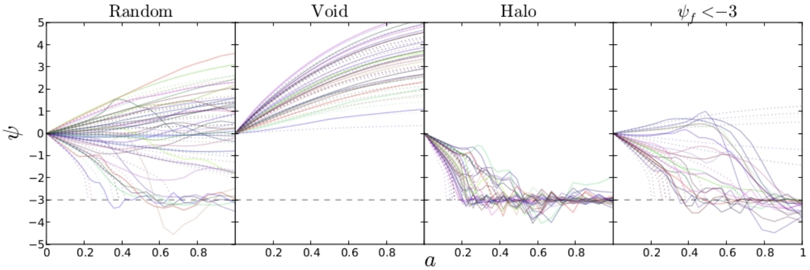
- An artifact in 2LPT: overdense spots in voids!
- The collapse "barrier": $\psi = -3$





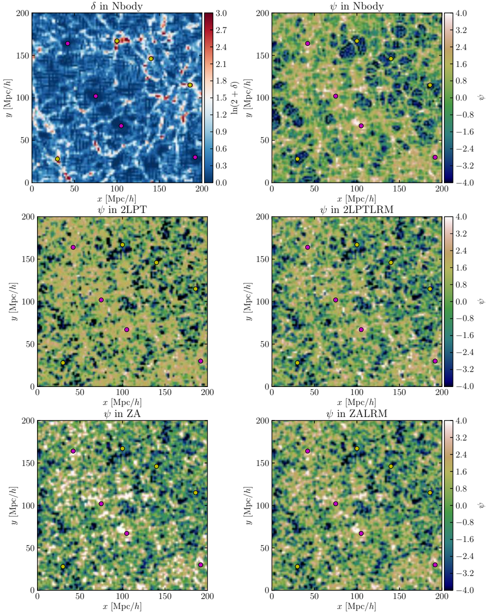
Further comments on ψ

- The collapse "barrier": $\psi = -3$



Neyrinck 2013

Further comments on ψ



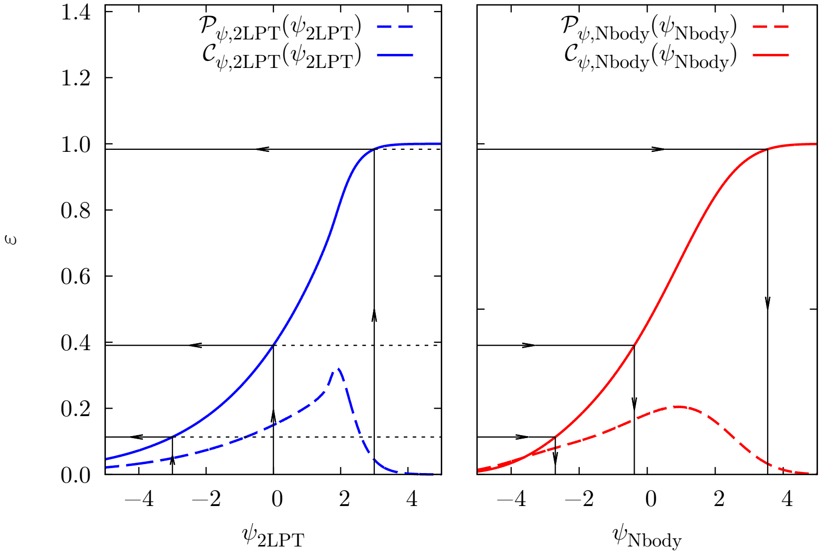
The Lagrangian remapping procedure

- Goal: Improve the correspondence between LPT-approximate models and full numerical *N*-body simulations of gravitational large-scale structure formation.
- Due to mode coupling, positive and negative fluctuations grow at different rates in the non-linear regime, but even non-linear evolution tends to preserve the *rank order* of the pixels, sorted by density.
- In Lagrangian description of cosmological large-scale structure, the divergence of the displacement field ψ plays a similar role as the Eulerian density contrast δ and is a more natural object.

$\Rightarrow$ Remapping algorithm:

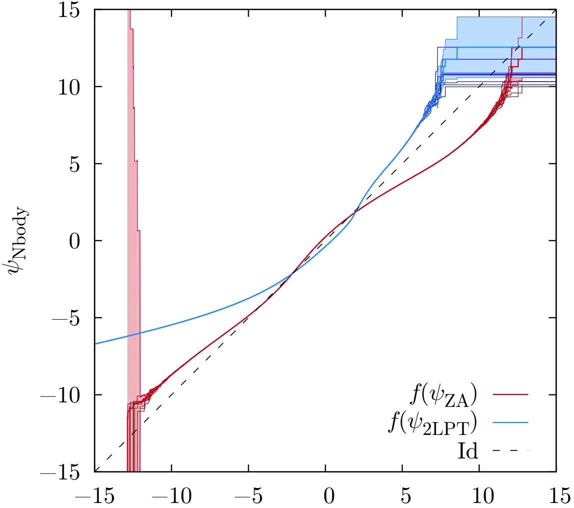
- keep positions of under- and over-densities predicted by LPT
- at pixel of rank order ψ_{LPT}, assign a new divergence of the displacement field, ψ_{Nbody}
- reconstruct the curl-free displacement field from its remapped divergence, and evolve the particles accordingly

The Lagrangian remapping procedure

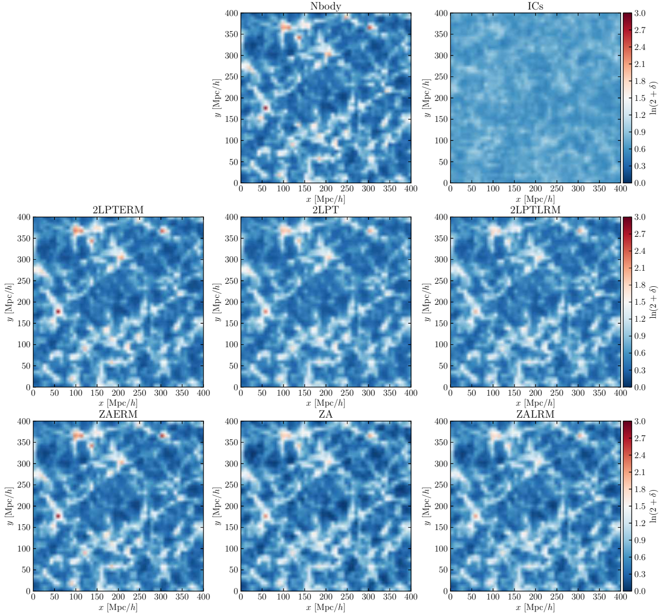


The Lagrangian remapping function

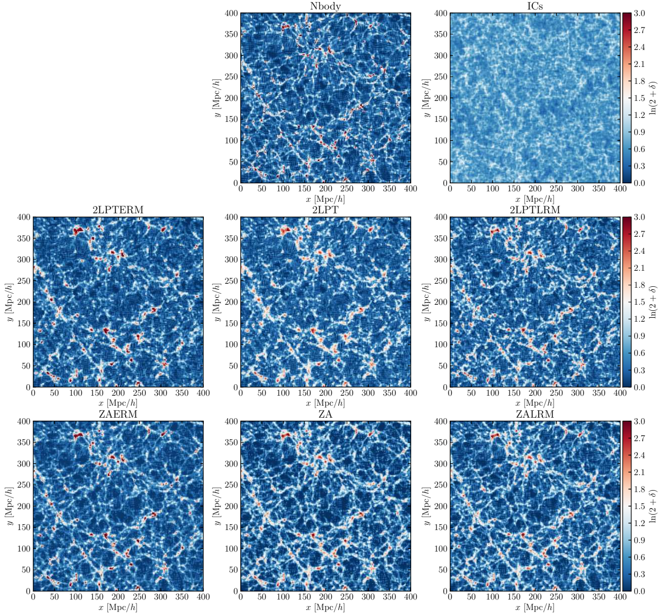
$$f(\psi_{\mathrm{LPT}}) \equiv \psi_{\mathrm{Nbody}} = \mathcal{C}_{\mathrm{Nbody}}^{-1}(\mathcal{C}_{\mathrm{LPT}}(\psi_{\mathrm{LPT}}))$$



Location of particles



Location of particles



Results: Two-point statistics

How does remapping affect the higher-order correlators?

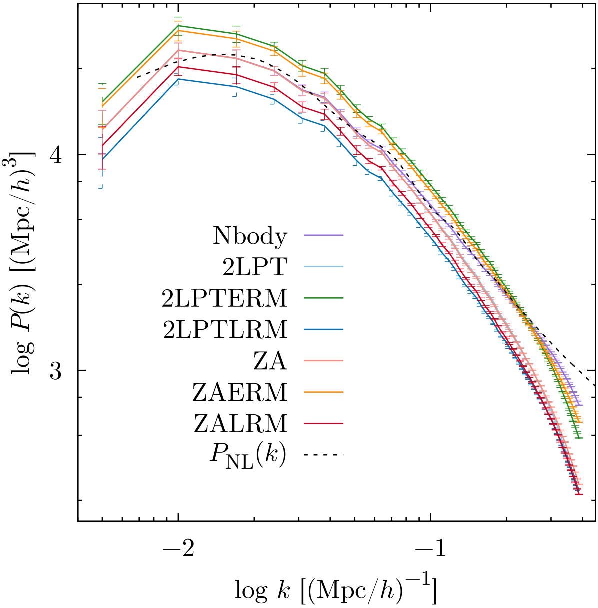
- we expect the higher-order correlations to be respected by the remapping procedure;
- possible improvements could be exploited in data analysis or artificial galaxy survey applications.

Power spectrum:

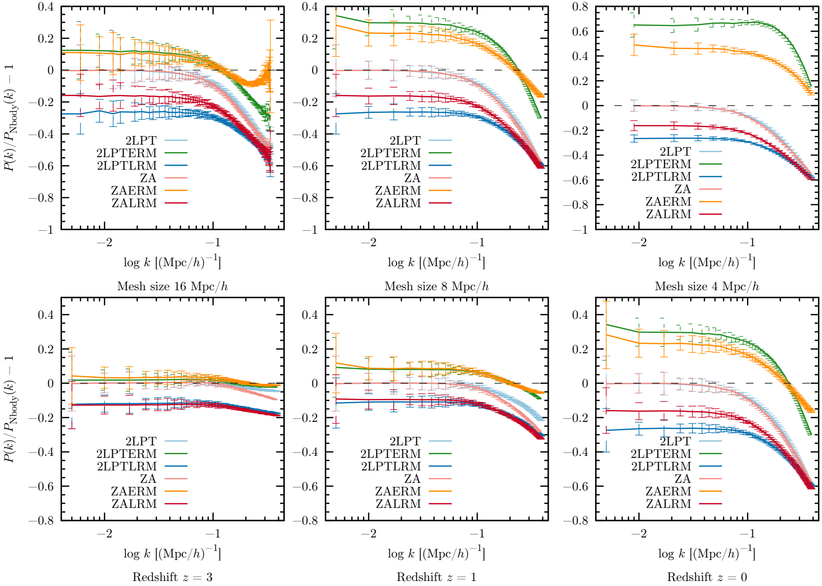
$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle = \delta_\mathrm{D}(\mathbf{k}_1 + \mathbf{k}_2)\, P(k)$$

- simplest statistic of interest beyond one-point function
- contains all information for a Gaussian random field (Wick's theorem)
- used in particular to derive the cosmological parameters

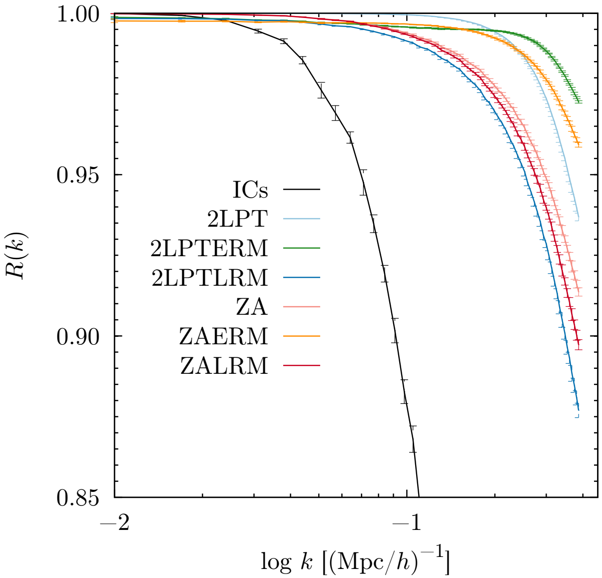
Power spectrum



Power spectrum: varying mesh size and redshift



Fourier-space cross-correlation coefficient

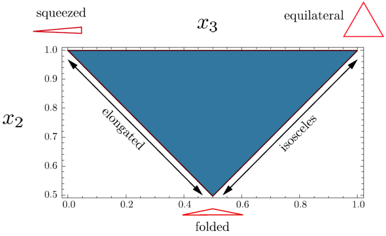


Results: Three-point statistics

Bispectrum:

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = \delta_{\mathrm{D}}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\, B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

- depends on triangle shape
- provides information on the galaxy bias (simplest model: $\delta_{\mathrm{g}} = b\,\delta_{\mathrm{m}}$)
- primordial non-Gaussianity probe



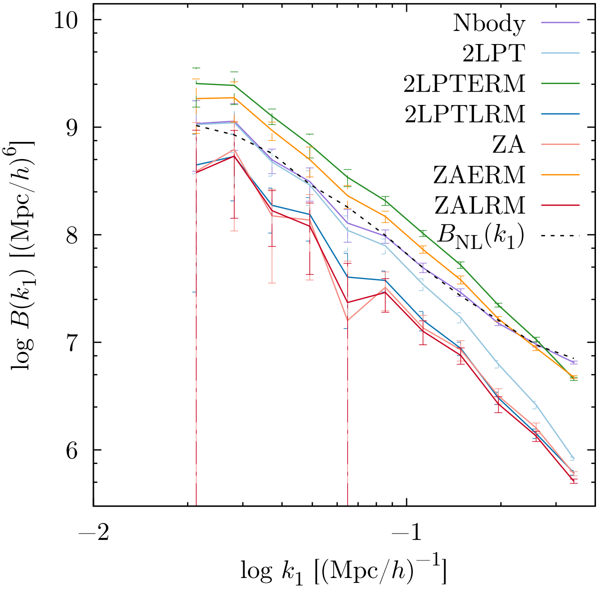
Redshift $z = 0$, mesh size 4 Mpc$/h$, folded triangles

Reduced bispectrum:

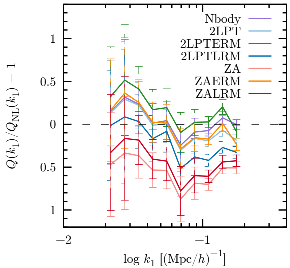
$$Q(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{P(k_1)P(k_2)+P(k_1)P(k_3)+P(k_2)P(k_3)}$$

takes away most of the dependence on scale and cosmology $\Rightarrow$ useful to isolate the effects of gravity (e.g. Gil-Marín *et al.* 2011, JCAP **11**, 019, arXiv:1109.2115)

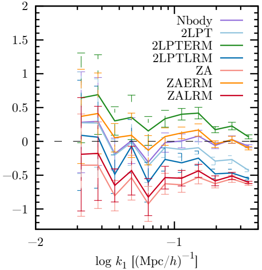
Bispectrum


10
9
8
7
6
−2
−1
log $B(k_1)$ [(Mpc/h)^6]
log k_1 [(Mpc/h)$^{-1}$]
Nbody
2LPT
2LPTERM
2LPTLRM
ZA
ZAERM
ZALRM
$B_{\mathrm{NL}}(k_1)$

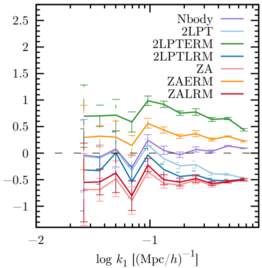
Bispectrum: varying mesh size and redshift



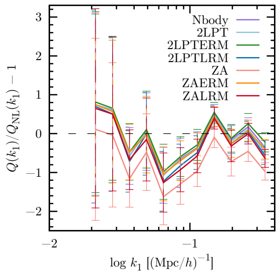
Mesh size 16 Mpc/h



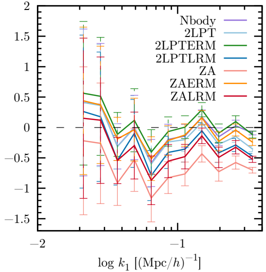
Mesh size 8 Mpc/h



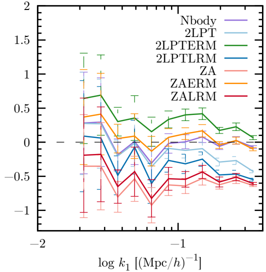
Mesh size 4 Mpc/h



Redshift $z = 3$

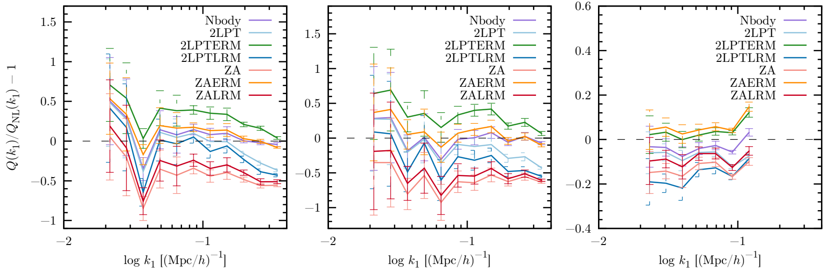


Redshift $z = 1$

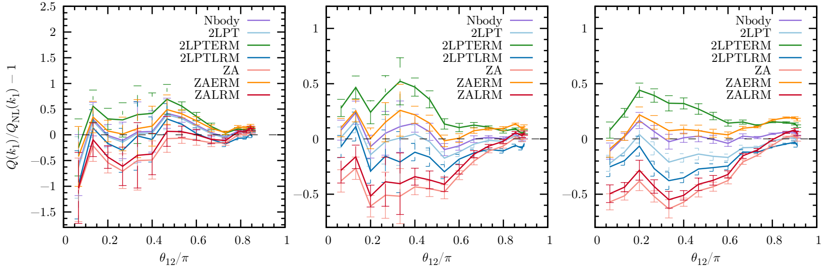


Redshift $z = 0$

Bispectrum: varying triangle shape



Squeezed triangles ($k_2 = k_1$, $\theta_{12} = \pi/6$)   Equilateral triangles ($k_2 = k_1$, $\theta_{12} = \pi/3$)   Folded triangles ($k_2 = 2k_1$, $\theta_{12} = 5\pi/6$)



$k_1 = k_2 = 0.05$ (Mpc/h)$^{-1}$ $k_1 = k_2 = 0.10$ (Mpc/h)$^{-1}$ $k_1 = k_2 = 0.15$ (Mpc/h)$^{-1}$

The cosmic web

What is the large-scale structure of the Universe made of?

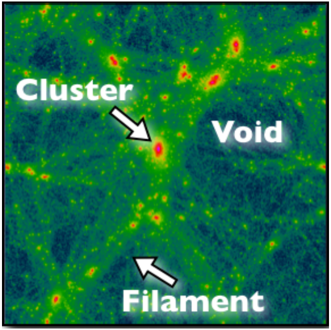


Figure: Courtesy of P. M. Sutter

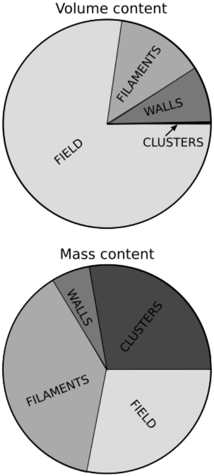


Figure: Aragón-Calvo, van de Weygaert & Jones, 2010

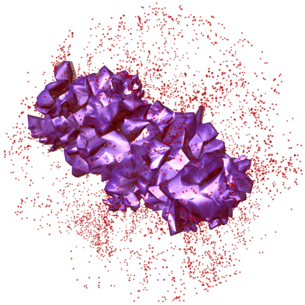
Cosmic voids in the large-scale structure of the Universe

What do we expect of voids?

- Number count:
  - cluster masses determination
  - void size determination
- Dynamics:
  - clusters are gravitationally collapsed objects, highly non-linear
  - voids can be found in the linear or mildy non-linear regime

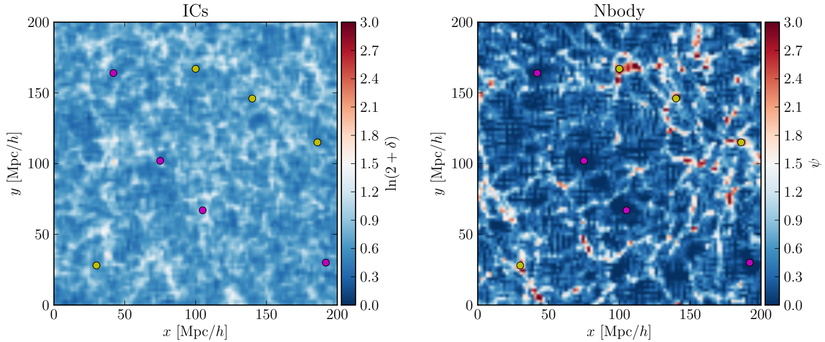
An efficient identification of voids is now possible thanks to numerical methods.

A public void catalog from the Sloan Digital Sky Survey DR7:



Sutter, Lavaux, Wandelt & Weinberg, 2012
http://www.cosmicvoids.net/

Dynamics of cosmic voids



Fundamental physics with cosmic voids

Some possible questions to be addressed with voids:

- relationship with the statistical properties of the ICs of the Universe
- relationship with the DM field and luminous tracers (the "bias" problem)
- tests of the standard GR picture of structure formation, discrimination among modified gravity models

First steps towards a systematic study of void statistics:

- The void one-point function (number count): provides constraints on the dark energy equation of state (Alizadeh, Biswas, Lavaux, Sutter, FL & Wandelt, in prep.)
- The void-void two-point correlation function: addresses the bias problem, the extraction of primordial non-Gaussianity (FL & Wandelt, in prep., Hamaus *et al.*, in prep.)

The void-void two-point correlation function in LPT

FL & Wandelt, in prep.

Correlations of $\left\langle \frac{1}{\rho} \times \frac{1}{\rho} \right\rangle$: puts weight on voids instead of clusters

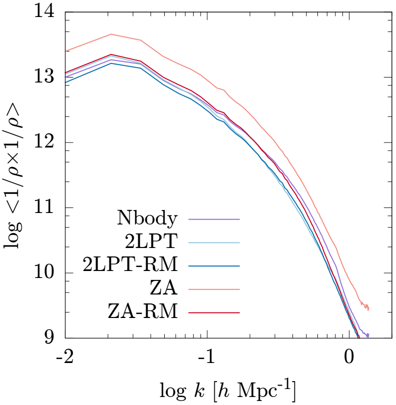


Figure: FL & Wandelt, preliminary

- the void-void correlation function can be modeled easily up to redshift zero using Lagrangian perturbation theory
- a Lagrangian remapping further improves the results at small scales or at low redshift

Concluding thoughts

Cosmic voids instead of galaxy clusters:

- simpler number count
- less affected by non-linearity
- earlier affected by dark energy

The remapping procedure: a fast way of producing mock galaxy distribution:

- A substantial improvement with respect to existing methods (NL affect even large scales: BAO: ~ 125 Mpc/h).
- Non-linear cosmological inference of the initial conditions of the Universe becomes feasible.

Outlook

- Constraints on primordial non-Gaussianities (f_{NL}) and therefore on inflationary models (multi-field inflation? non-standard kinetic term? periods of fast-roll? non-trivial pre-inflationary state? non-Bunch-Davies vacuum?).
- Inference of the initial conditions and of the properties of dark energy with cosmic voids statistics