Bayesian large-scale structure inference: initial conditions and the cosmic web

Florent Leclercq

Institut d'Astrophysique de Paris
Institut Lagrange de Paris
École polytechnique ParisTech

May 27th, 2014

In collaboration with:

Jacopo Chevallard (U. São Paulo), Nico Hamaus (IAP), Jens Jasche (IAP), Guilhem Lavaux (IAP), Emilio Romano-Díaz (U. Bonn), Paul M. Sutter (IAP/Ohio State U.), Alice Pisani (IAP), Benjamin Wandelt (IAP/U. Illinois)

A joint problem

- **How did the Universe begin?**
  - What are the statistical properties of the initial conditions?

- **How did structure appear in the Universe?**
  - What is the physics of dark matter and dark energy?

- Usually these problems are addressed in isolation.
- This talk:
  - A case for physical inference of four-dimensional dynamic states
  - A description of methodology and progress towards enriching the standard for analysis of galaxy surveys

Why Bayesian inference?

- Why do we need Bayesian inference?

Inference of signals = ill-posed problem

- Incomplete observations: survey geometry, selection effects
- Cosmic variance
- Noise, biases, systematic effects

⇨ **No unique recovery is possible!**

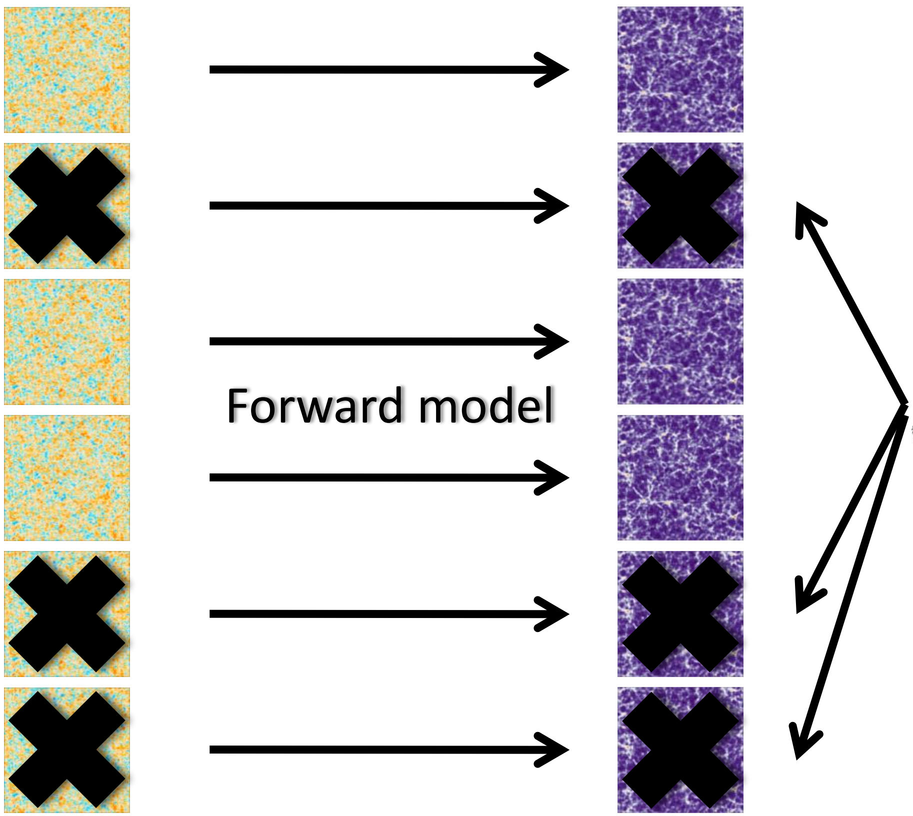
"What are the initial conditions of the Universe?" ⇨ "What is the probability distribution of possible initial conditions (signals) compatible with the observations?"

"What is the shape of the cosmic web in the local Universe?" ⇨ "What is the probability distribution of possible web-types (signals) compatible with the observations?"

$$p(s|d)p(d) = p(d|s)p(s)$$

Bayesian forward modeling: the ideal scenario

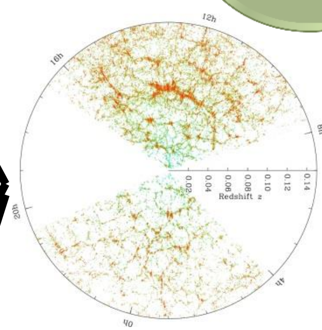
Forward model = N-body simulation + Halo occupation + Galaxy formation + Feedback + …



All possible ICs

All possible FCs

$d \approx 10^7$

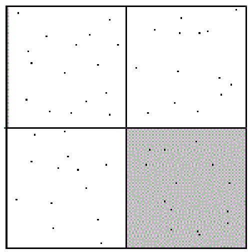


Observations

We need a very, very, very big computer!

(Parameter) Space: the final frontier

- The "curse of dimensionality" Bellman 1961

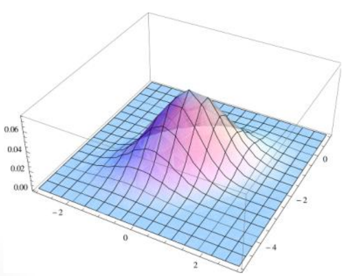


dimension	fraction of particles in quadrant of hypercube
1	$2^{-1} = 0.5$
10	$2^{-10} = 9.7 \times 10^{-4}$
100	$2^{-100} = 7.8 \times 10^{-31}$
1000	$2^{-1000} = 9.3 \times 10^{-302}$

Adding extra dimensions…

- Exponential increase of the number of particles needed for uniform sampling
- Exponential increase of sparsity given a fixed amount of particles

- High-dimensional probability distribution functions



→ Traditional sampling methods will fail

but gradients carry capital information

Hamiltonian Monte Carlo

- Use classical mechanics to solve statistical problems!
  - The potential: $\psi(\mathbf{x}) \equiv -\ln(\mathcal{P}(\mathbf{x}))$
  - The Hamiltonian: $H \equiv \frac{1}{2}\mathbf{p}^T\mathbf{M}^{-1}\mathbf{p} + \psi(\mathbf{x})$

$$(\mathbf{x},\mathbf{p}) \Rightarrow \left\{ \begin{aligned} \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} &= \frac{\partial H}{\partial \mathbf{p}} = \mathbf{M}^{-1}\mathbf{p} \\ \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} &= -\frac{\partial H}{\partial \mathbf{x}} = -\frac{\partial \psi(\mathbf{x})}{\partial \mathbf{x}} \end{aligned} \right\} \Rightarrow (\mathbf{x}',\mathbf{p}')$$

gradients

$$a(\mathbf{x}',\mathbf{x}) = \mathrm{e}^{-(H'-H)} = 1$$

acceptance ratio unity

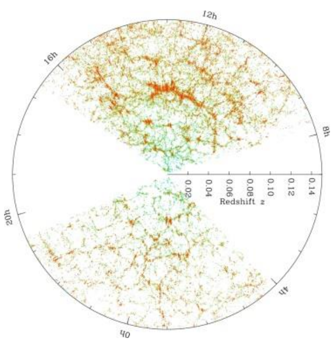
- HMC beats the curse of dimensionality by:
  - Exploiting gradients
  - Using conservation of Hamiltonian

Duane *et al.* 1987

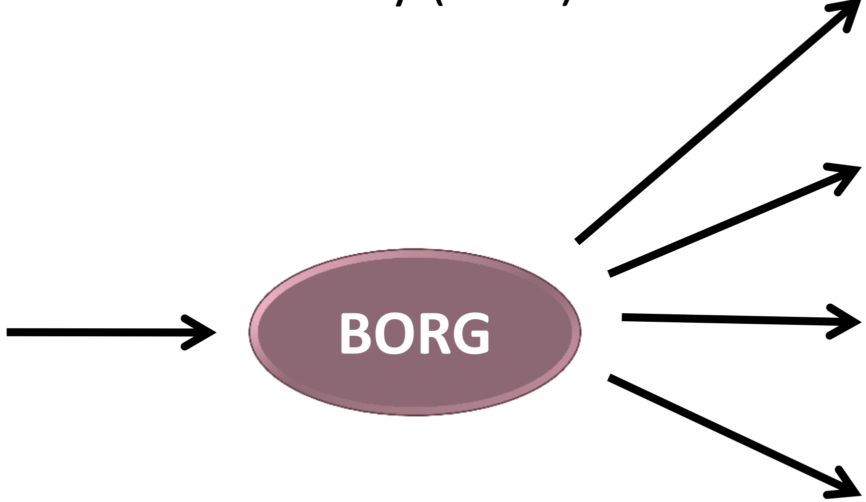
BORG: *Bayesian Origin Reconstruction from Galaxies*

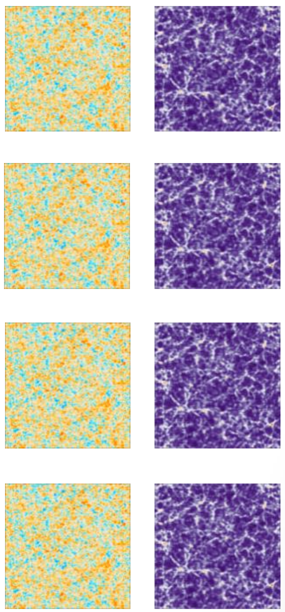
What makes the problem tractable:

- **Sampler**: Hamiltonian Markov Chain Monte Carlo method
- **Physical model**: Second-order Lagrangian perturbation theory (2LPT)



Observations





Samples of possible 4D states

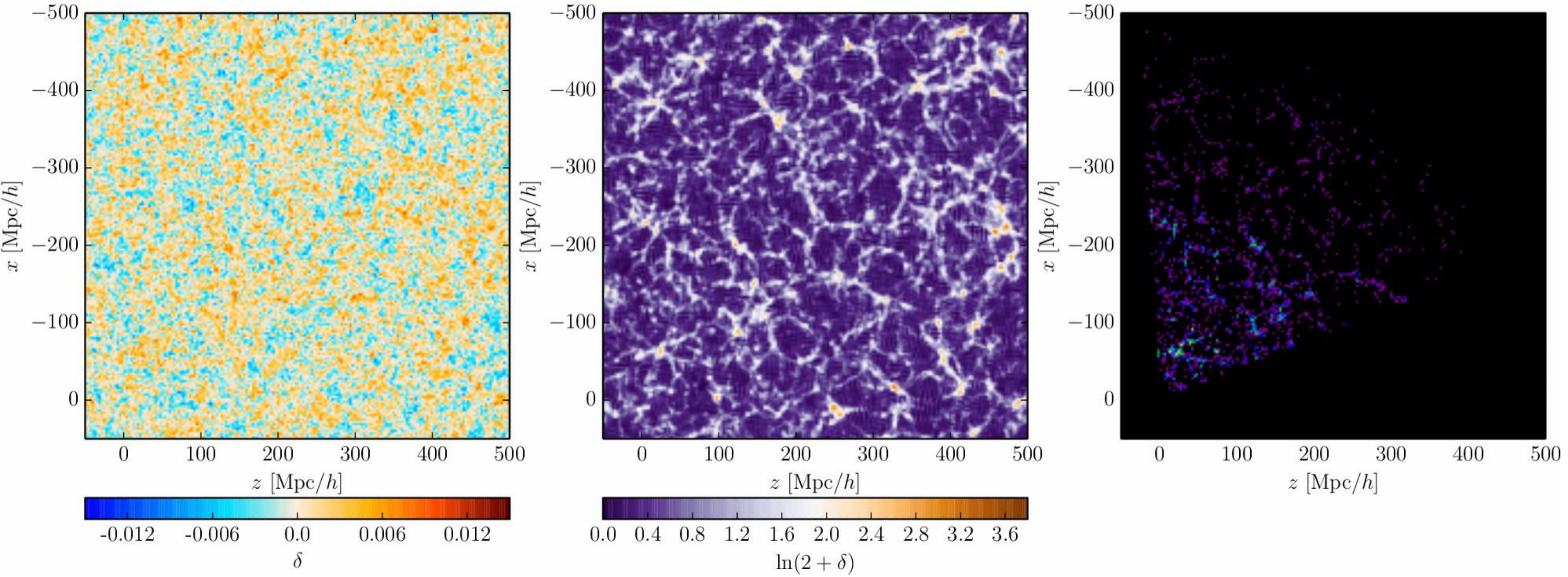
Jasche & Wandelt 2013, arXiv:1203.3639

The BORG SDSS run

- 463,230 galaxies from the NYU-VAGC based on SDSS DR7
- Comoving cubic box of side length 750 Mpc/h, with periodic boundary conditions
- 256^3 grid, resolution 3 Mpc/h ⇒ ≈ 17 millions parameters
- 10,000 samples, four-dimensional maps
- ≈ 3 TB disk space
- 8 months wallclock time on 16-32 cores (and still running!)

Jasche, FL & Wandelt, in prep.

BORG at work

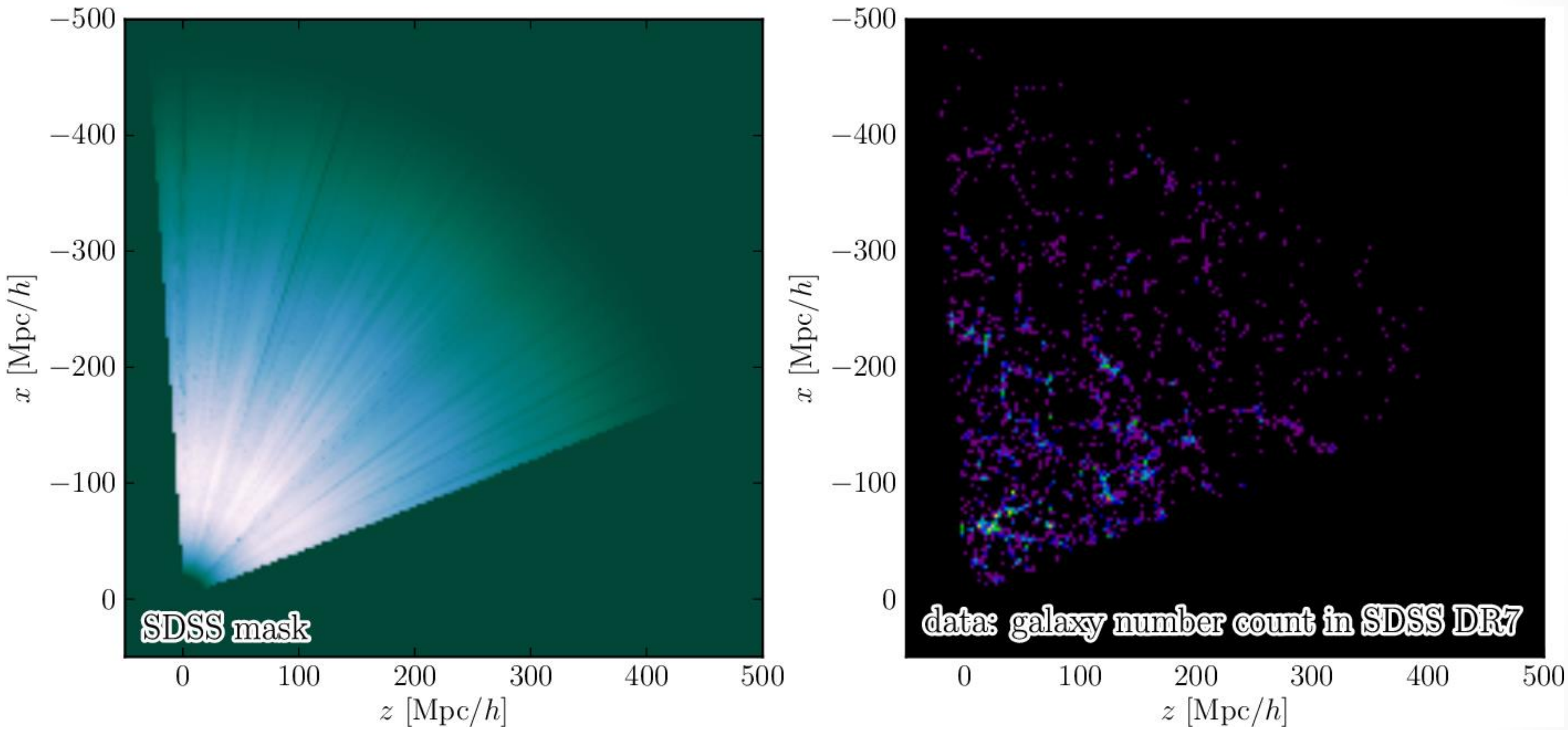


Initial conditions

Final conditions

Observations

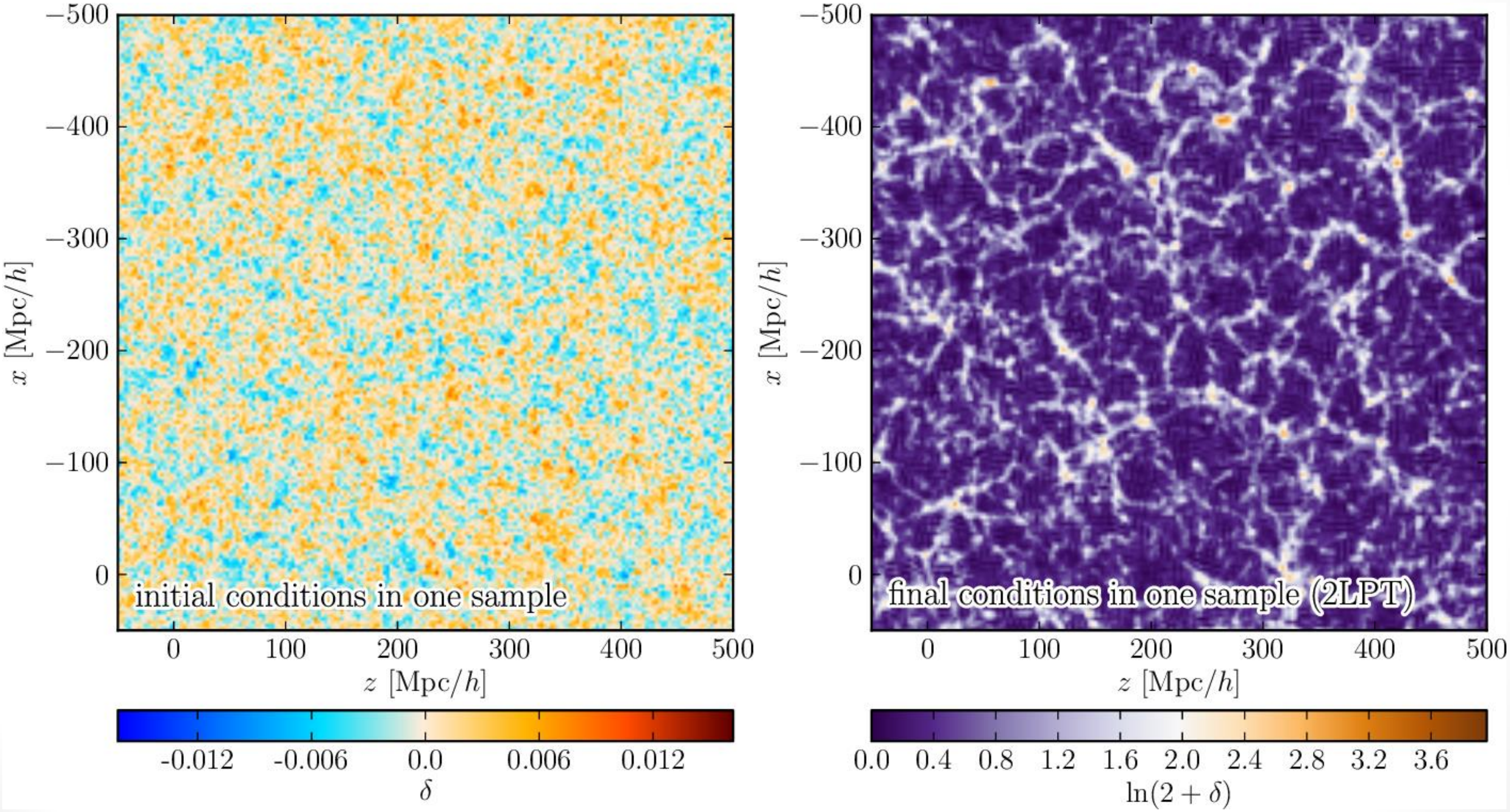
Bayesian chronocosmography from SDSS DR7



Jasche, FL & Wandelt, in prep.

Data

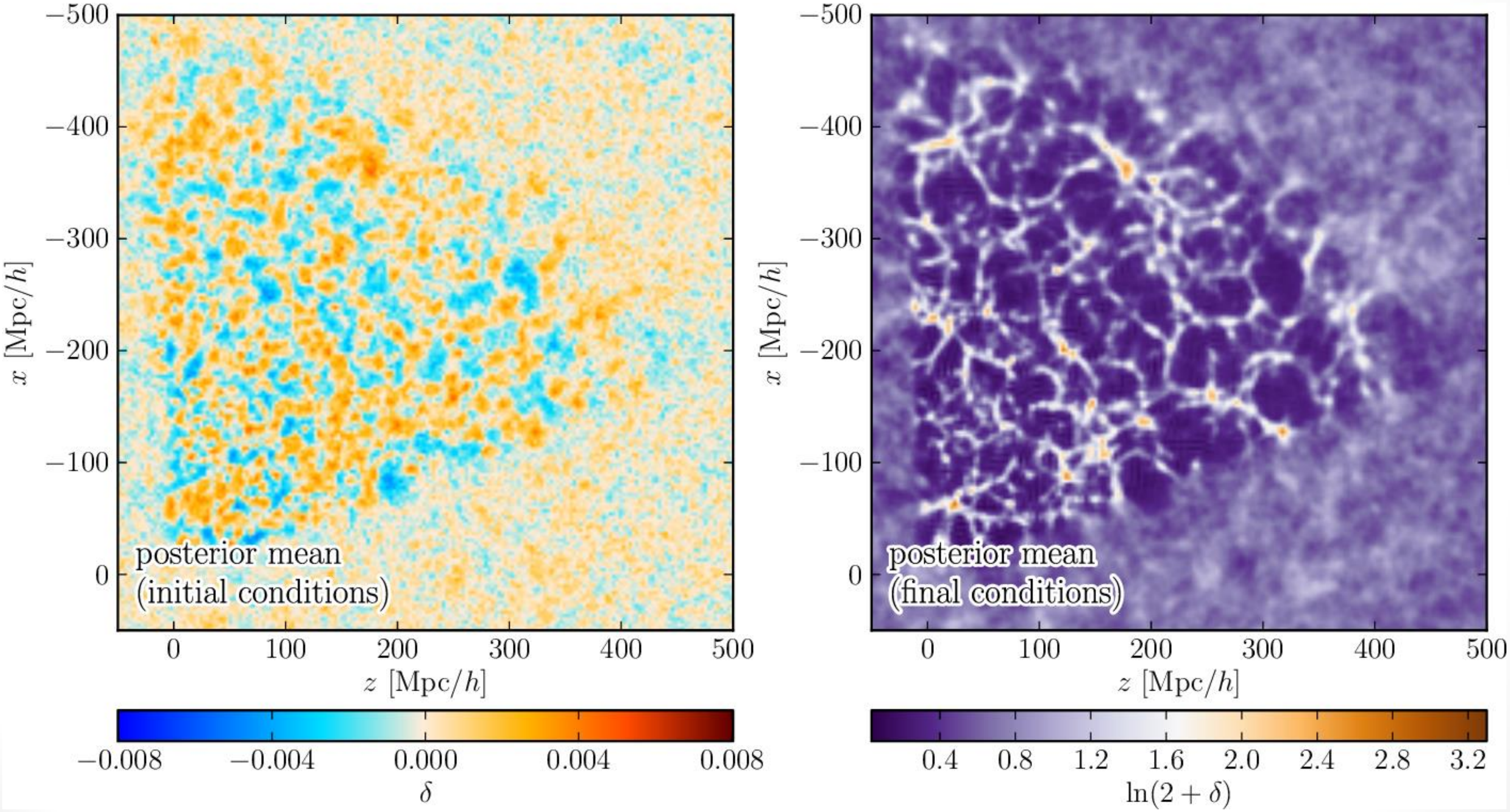
Bayesian chronocosmography from SDSS DR7



One sample

Jasche, FL & Wandelt, in prep.

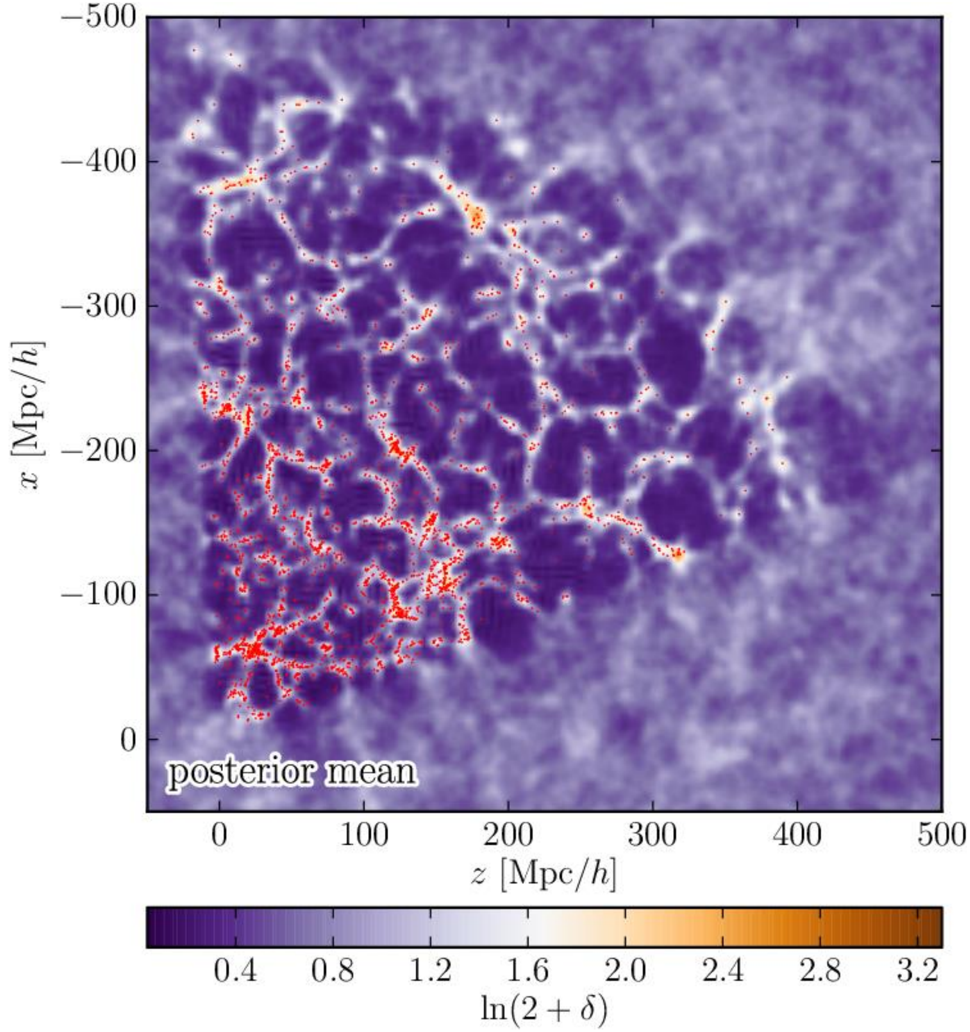
Bayesian chronocosmography from SDSS DR7



Posterior mean

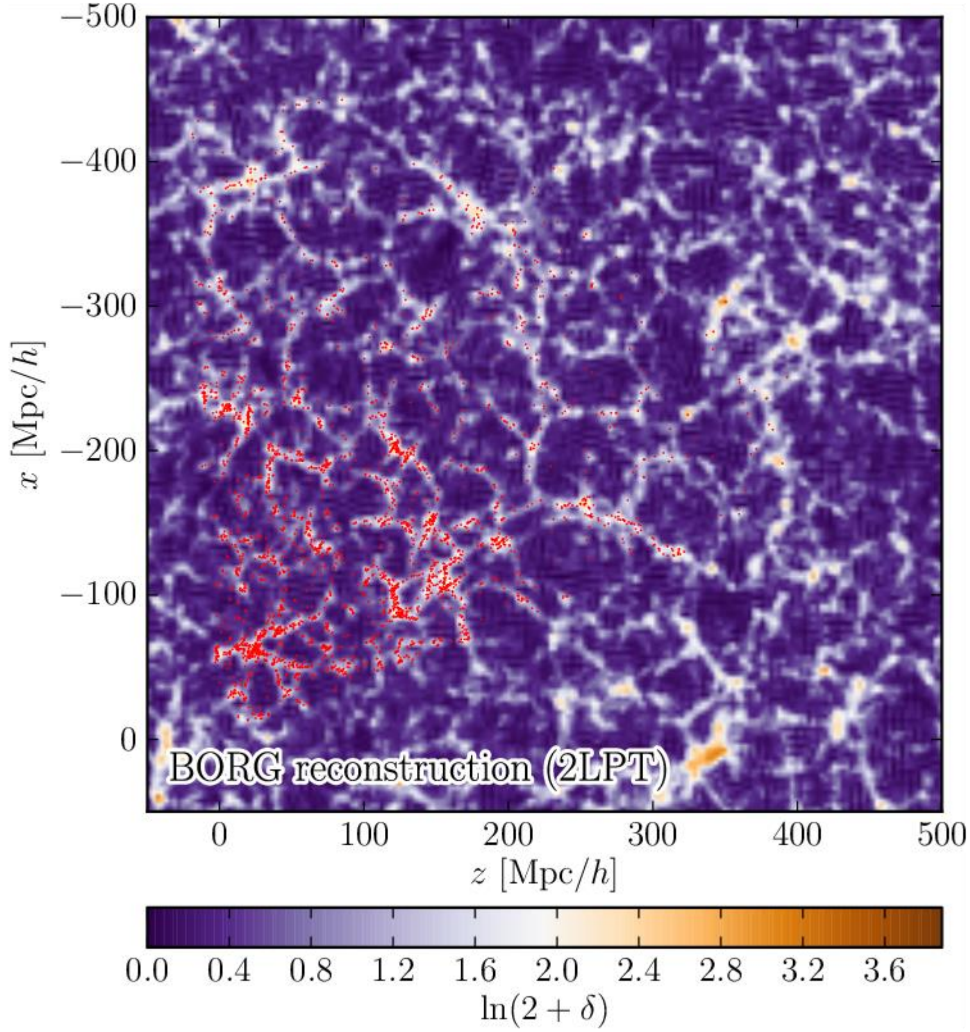
Jasche, FL & Wandelt, in prep.

Bayesian chronocosmography from SDSS DR7



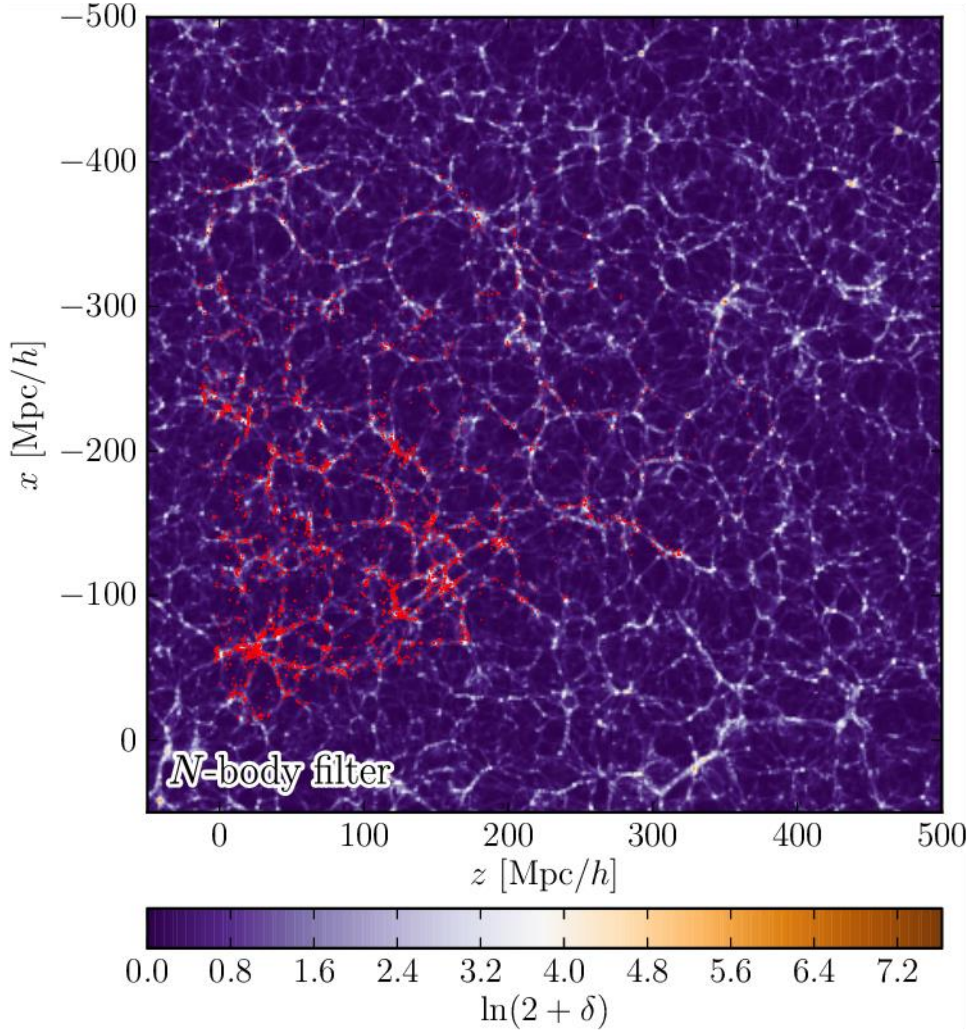
Jasche, FL & Wandelt, in prep.

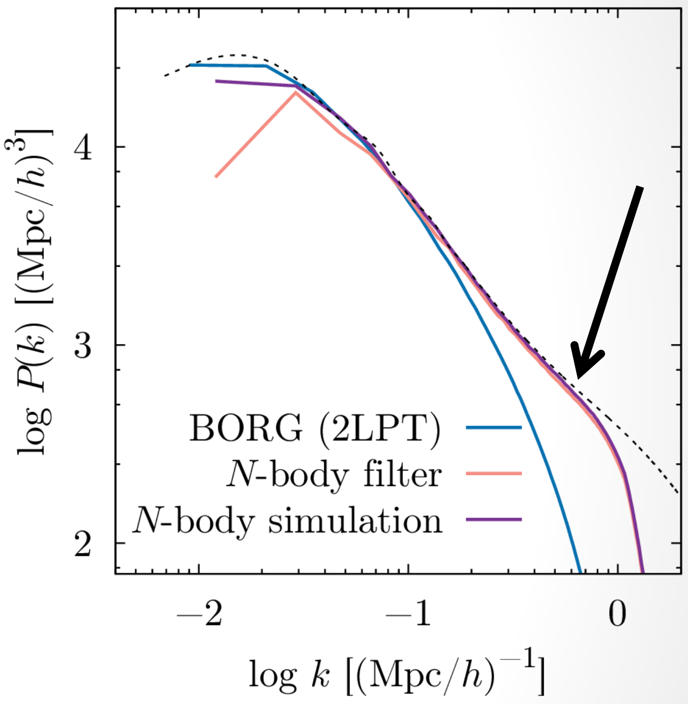
Non-linear filtering



Jasche, FL, Romano-Diaz & Wandelt, in prep.

Non-linear filtering



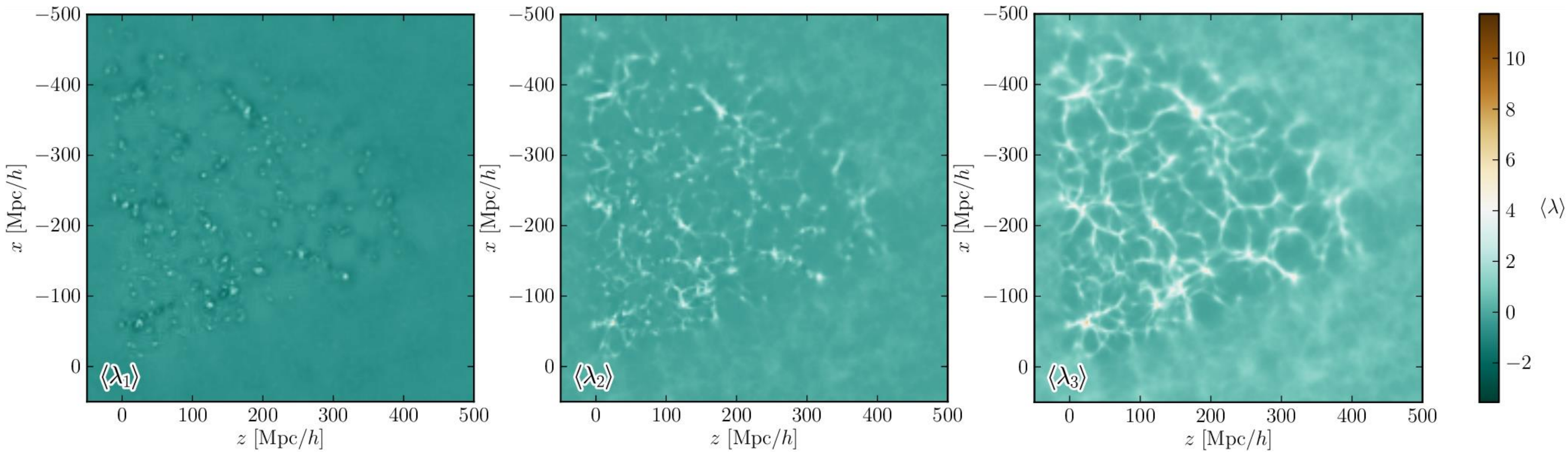


Jasche, FL, Romano-Diaz & Wandelt, in prep.

Tidal shear analysis

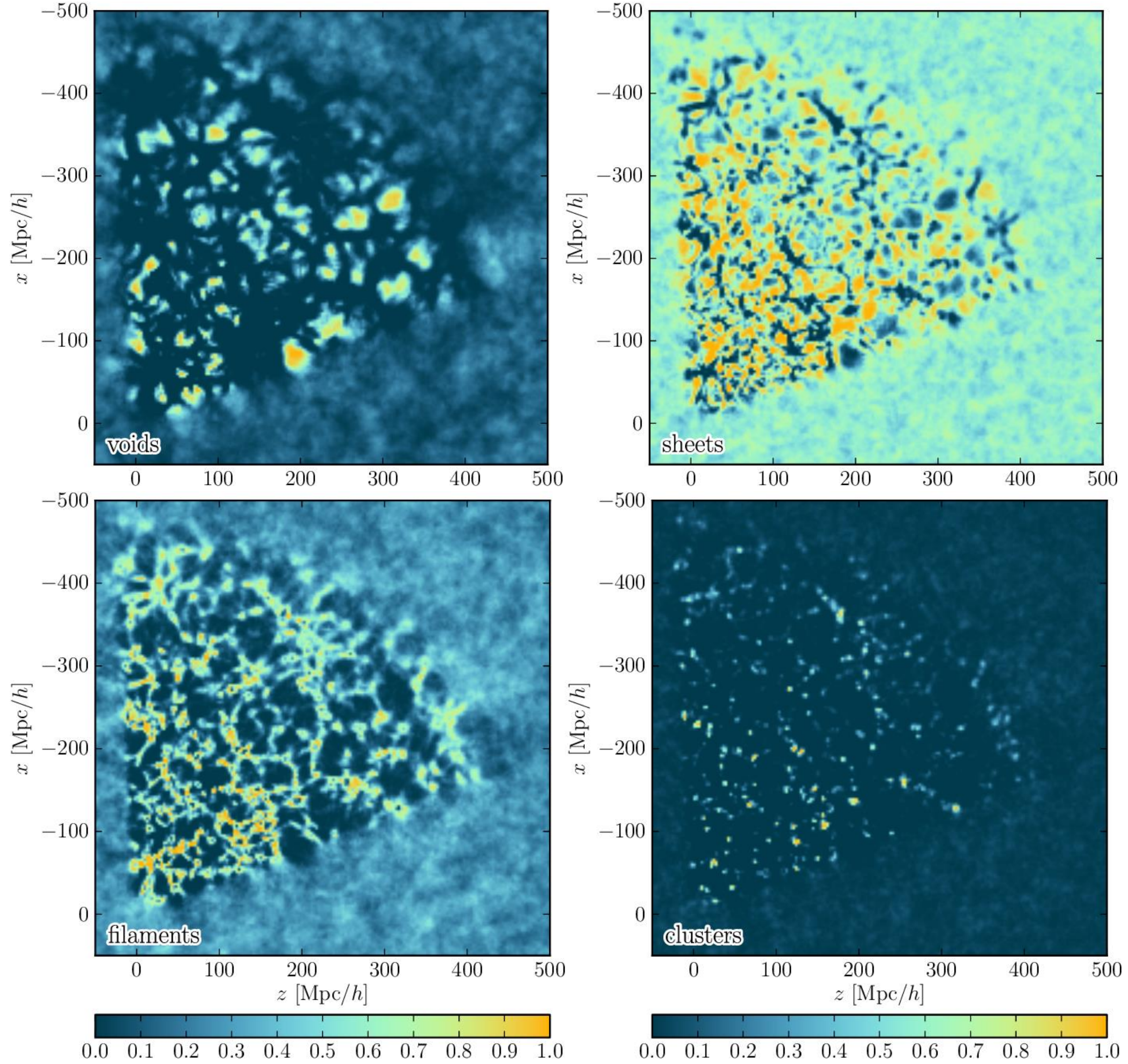
- $\lambda_1, \lambda_2, \lambda_3$: eigenvalues of the tidal field tensor, the Hessian of the gravitational potential: $T_{ij} = \partial_i \partial_j \Phi$
  - Voids: $\lambda_1, \lambda_2, \lambda_3 < 0$
  - Sheets: $\lambda_1 > 0$ and $\lambda_2, \lambda_3 < 0$
  - Filaments: $\lambda_1, \lambda_2 > 0$ and $\lambda_3 < 0$
  - Clusters: $\lambda_1, \lambda_2, \lambda_3 > 0$

Hahn, Porciani, Carollo & Dekel, 2006, arXiv:astro-ph/0610280



FL, Jasche, Chevallard & Wandelt, in prep.

Dynamic structures inferred by BORG

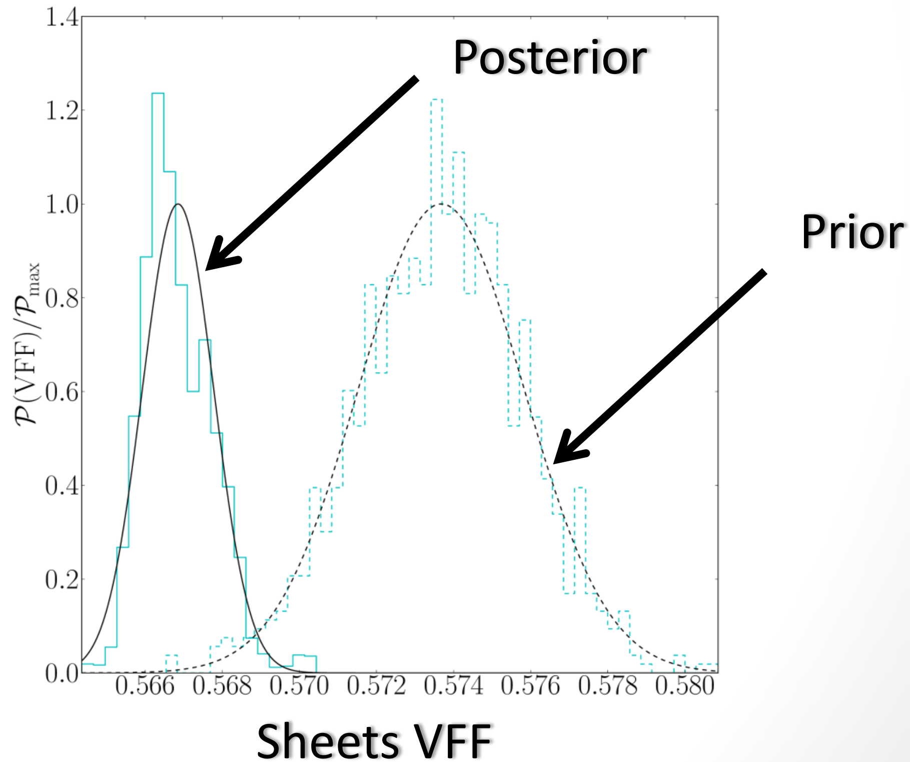


FL, Jasche, Chevallard & Wandelt, in prep.

Volume filling fractions

Structure type	μ_{VFF}	σ_{VFF}	μ_{VFF}	σ_{VFF}
	Posterior		Prior	
Void	0.14434	8.9091×10^{-4}	0.14164	6.1171×10^{-3}
Sheet	0.56685	9.2314×10^{-4}	0.57368	2.1466×10^{-3}
Filament	0.26127	7.6349×10^{-4}	0.26081	4.1730×10^{-3}
Halo	0.02753	1.3551×10^{-4}	0.02389	4.4993×10^{-4}

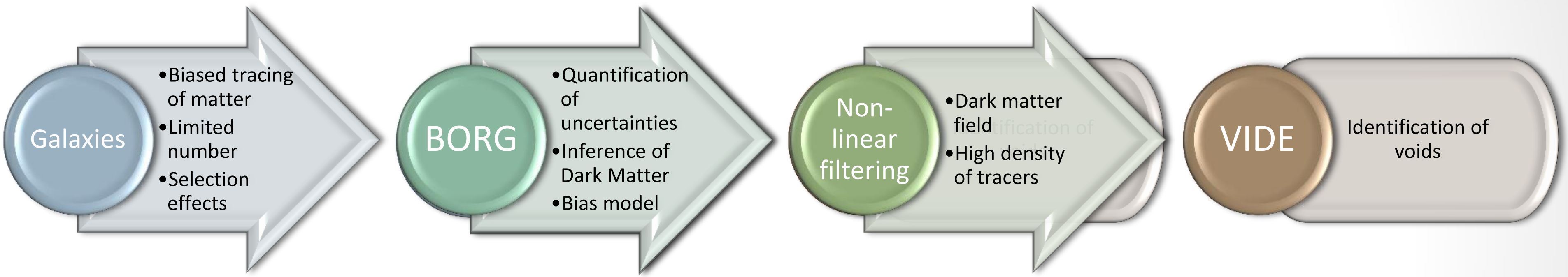
- Bayesian reasoning!
  - Full pdf
  - Update of the state of knowledge



FL, Jasche, Chevallard & Wandelt, in prep.

Dark matter voids in the SDSS

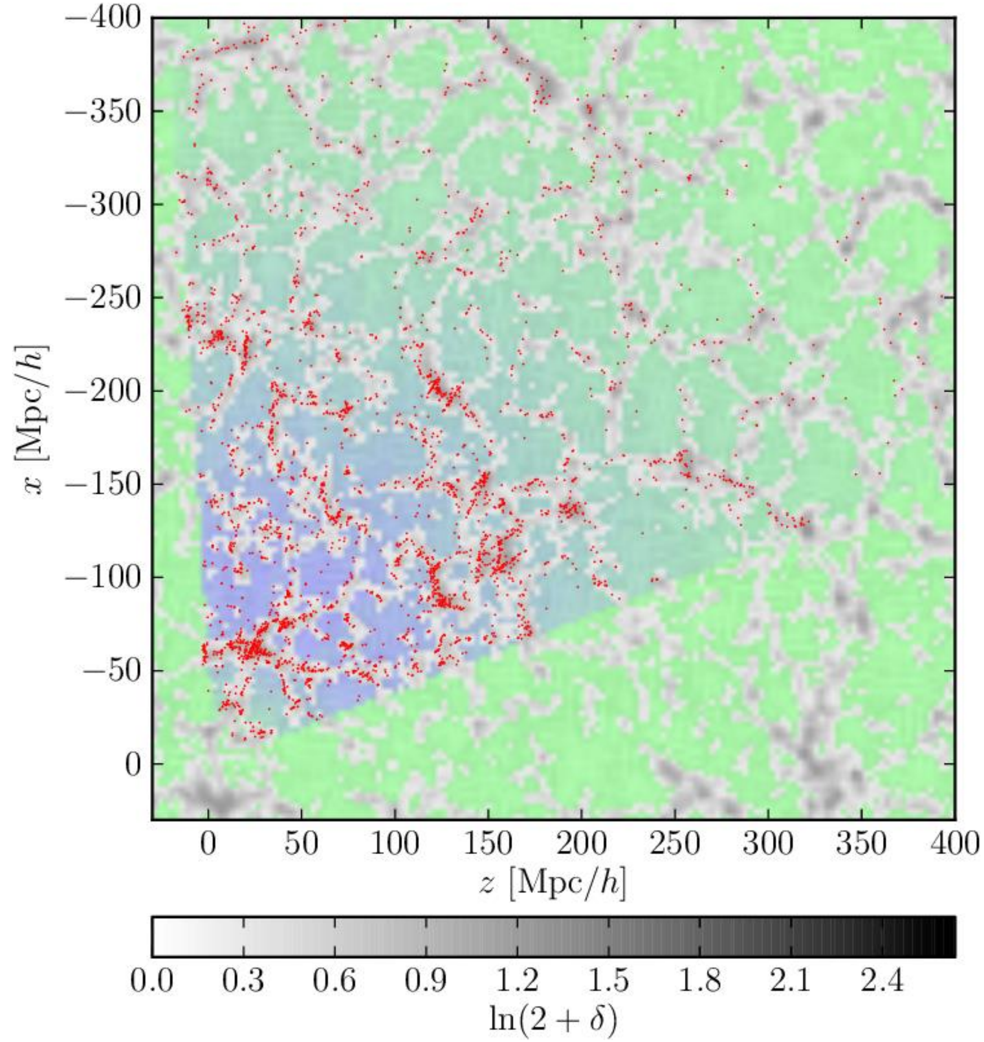
- How?



- Why? What is made possible by our technology:
  - **Bias**. Voids are defined in the dark matter distribution, not in galaxies.
  - **Shot noise**. Galaxies sparsely sample the dark matter distribution. We get 10x more dark matter voids than galaxy voids.

FL, Jasche, Sutter, Hamaus, Lavaux, Pisani & Wandelt, in prep.

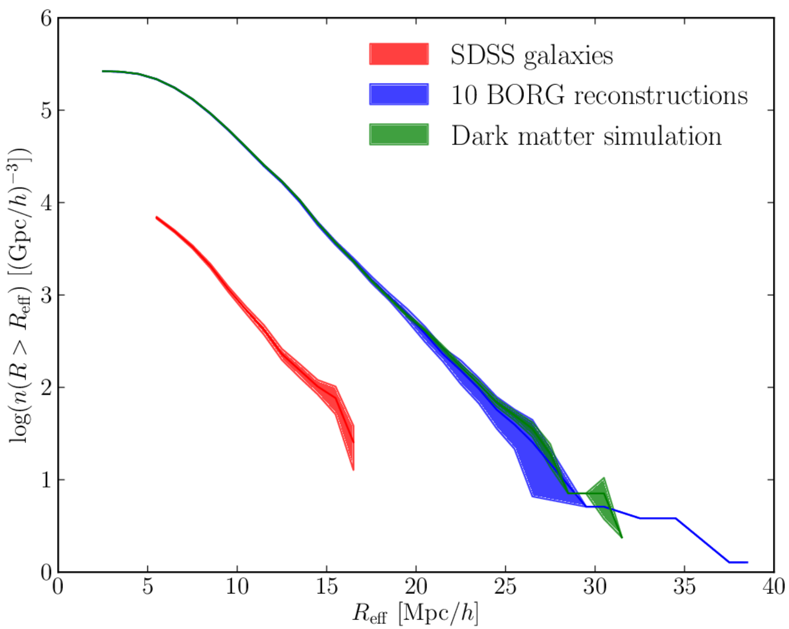
Dark matter voids in the SDSS



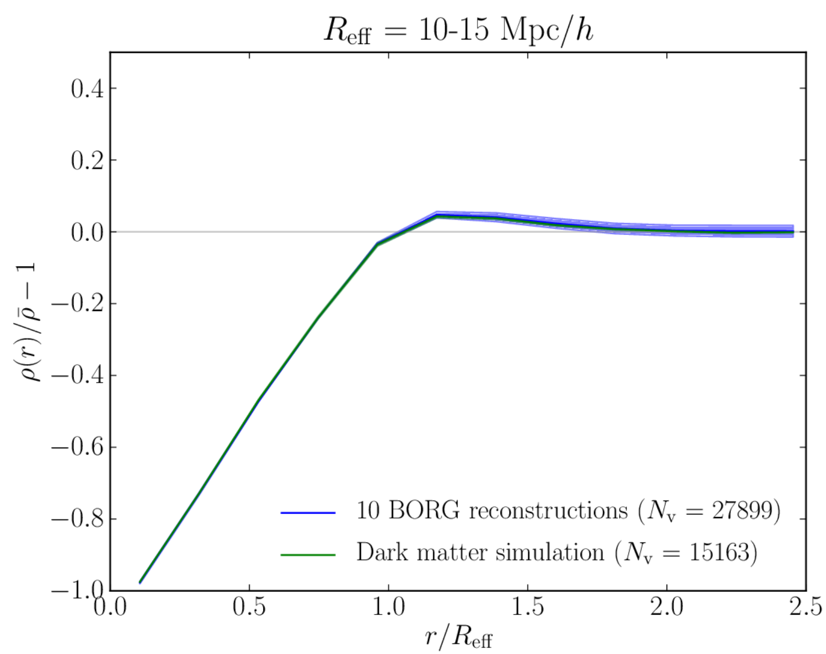
FL, Jasche, Sutter, Hamaus, Lavaux, Pisani & Wandelt, in prep.

Properties of dark matter voids

- For all usual void statistics, results are **consistent with *N*-body simulations** prepared with the same setup



void number count



void radial density profile

FL, Jasche, Sutter, Hamaus, Lavaux, Pisani & Wandelt, in prep.

Summary & Conclusions

- Bayesian large-scale structure inference in 10 millions dimensions is possible!
  - Non-linear and non-Gaussian inference
  - Uncertainty quantification (noise, survey geometry, selection effects and biases)

- Application to data: four-dimensional chronocosmography
  - Physical reconstruction of the initial conditions
  - Characterization of the cosmic web in the local Universe