How is the cosmic web woven?

A Bayesian approach

Florent Leclercq

Institute of Cosmology and Gravitation, University of Portsmouth

http://icg.port.ac.uk/~leclercq/

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In collaboration with:

Jens Jasche (ExC Universe, Garching), Guilhem Lavaux (IAP),

Will Percival (ICG), Benjamin Wandelt (IAP/U. Illinois)

How is the cosmic web woven?

A joint problem!

- **How did the Universe begin?**
  - What are the statistical properties of the initial conditions?

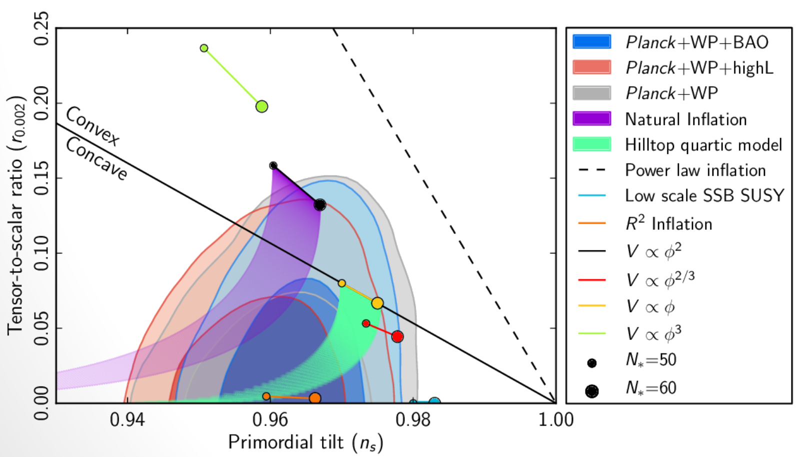
- **How did the large-scale structure take shape?**
  - What is the physics of dark matter and dark energy?

We have theoretical and computer models…

- **Initial conditions:**
  a Gaussian random field

$$\mathcal{P}(\delta^{\mathrm{i}}|S) = \frac{1}{\sqrt{|2\pi S|}} \exp\left(-\frac{1}{2}\sum_{x,x'} \delta^{\mathrm{i}}_x S^{-1}_{xx'} \delta^{\mathrm{i}}_{x'}\right)$$

Everything seems consistent with the simplest inflationary scenario, as tested by Planck.



Planck 2015 XX, arXiv:1502.02114

- **Structure formation:**
  numerical solution of the Vlasov-Poisson system for dark matter dynamics

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{ma} \cdot \nabla f - ma\nabla\Phi \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$

$$\Delta\Phi = 4\pi \mathrm{G} a^2 \bar{\rho}\delta$$

Y. Dubois & S. Colombi (IAP)

But some questions remain

1. How do we **test** these frameworks?
   - Usually the two problems of initial conditions and structure formation are addressed in isolation.
   - Ideally, galaxy surveys should be analyzed in terms of the joint constraints that they place on these two questions.

2. How did this happen in **our** Universe?

1. How do we test our models?

J. Cham - PhD comics

Redshift range	Volume (Gpc^3)	k_{max} $(Mpc/h)^{-1}$	N_{modes}
0-1	50	0.15	10^7
1-2	140	0.5	$5x10^8$
2-3	160	1.3	10^{10}

M. Zaldarriaga

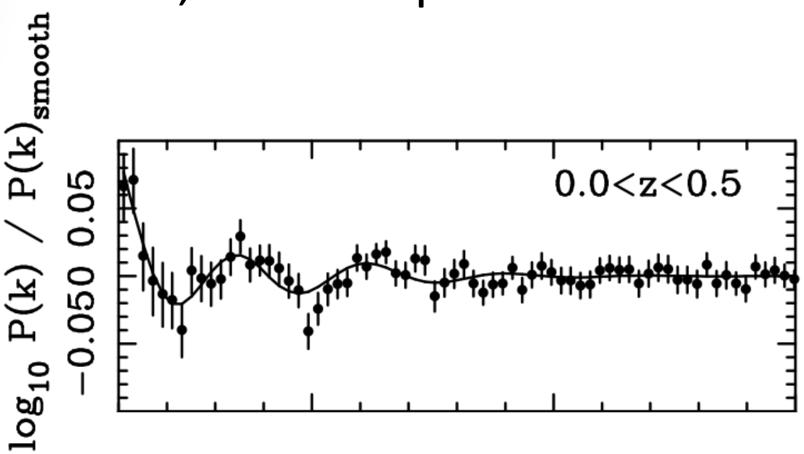
- Precise tests require many modes.
- In 3D galaxy surveys, the number of modes usable scales as k^3_{max}.

- The challenge: non-linear evolution at **small scales** and **late times**.
- The strategy:
  - Pushing down the smallest scale usable for cosmological analysis
  - Inferring the initial conditions from galaxy positions

In other words: go beyond the **linear** and **static** analysis of the LSS.

2. How did this happen in our Universe?

- This means that we cannot do, for example:



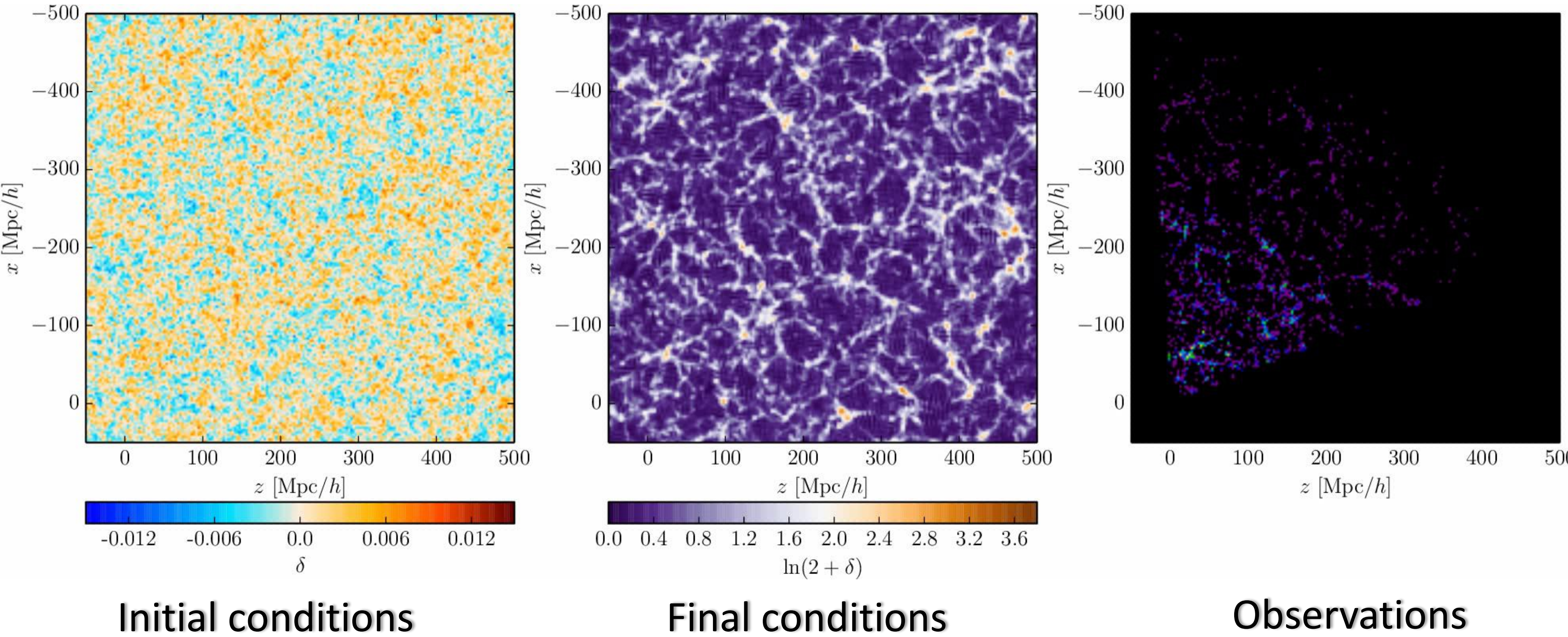
Percival *et al.* 2010, arXiv:0907.1660

- Standard analyses: reduce the data to some statistics, then fit some model parameters

- We have to do a **joint analysis** of all aspects, including **density reconstruction**
  - Provides powerful constraints
  - Propagates uncertainties between all parts of the analysis
  - Avoids using the data twice
- It is a process known as **data assimilation**

Can we just **fit the entire survey**?

SDSS chrono-cosmography



The BORG SDSS run:

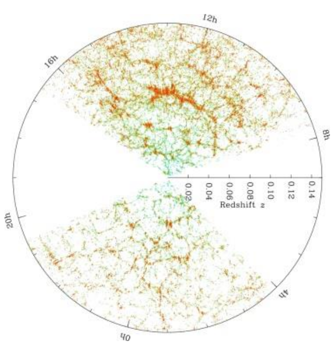
334,074 galaxies, ≈ 17 millions parameters, 12,000 samples, 3 TB, 10 months on 32 cores

Jasche, FL & Wandelt 2015, arXiv:1409.6308

How did we get that?

BORG: *Bayesian Origin Reconstruction from Galaxies*

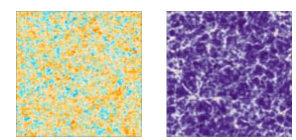
- **Data model**: Gaussian prior – Second-order Lagrangian perturbation theory (2LPT) – Poisson likelihood
  (and also: luminosity-dependent galaxy bias, automatic noise level calibration)
- **Sampler**: Hamiltonian Markov Chain Monte Carlo method



Observations

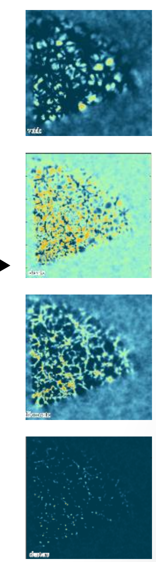
(galaxy catalog + meta-data: selection functions, completeness…)

→ BORG →



Inferred dark matter density

→



Cosmic web analysis

Jasche & Wandelt 2013, arXiv:1203.3639

Jasche, FL & Wandelt 2015, arXiv:1409.6308

COLA: *COmoving Lagrangian Acceleration*

- Write the displacement vector as: $\mathbf{s} = \mathbf{s}_{\mathrm{LPT}} + \mathbf{s}_{\mathrm{MC}}$

Tassev & Zaldarriaga 2012, arXiv:1203.5785

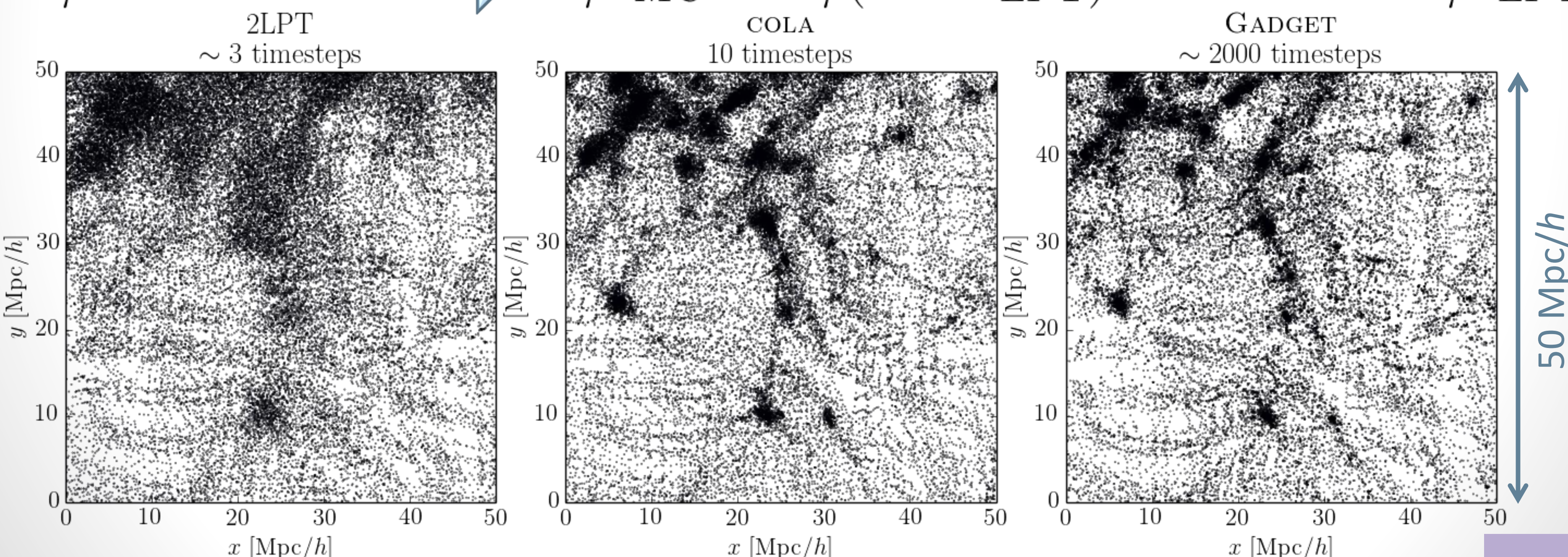
- Time-stepping (omitted constants and Hubble expansion):

Standard:

$$\partial_\tau^2 \mathbf{s} = -\nabla\Phi$$

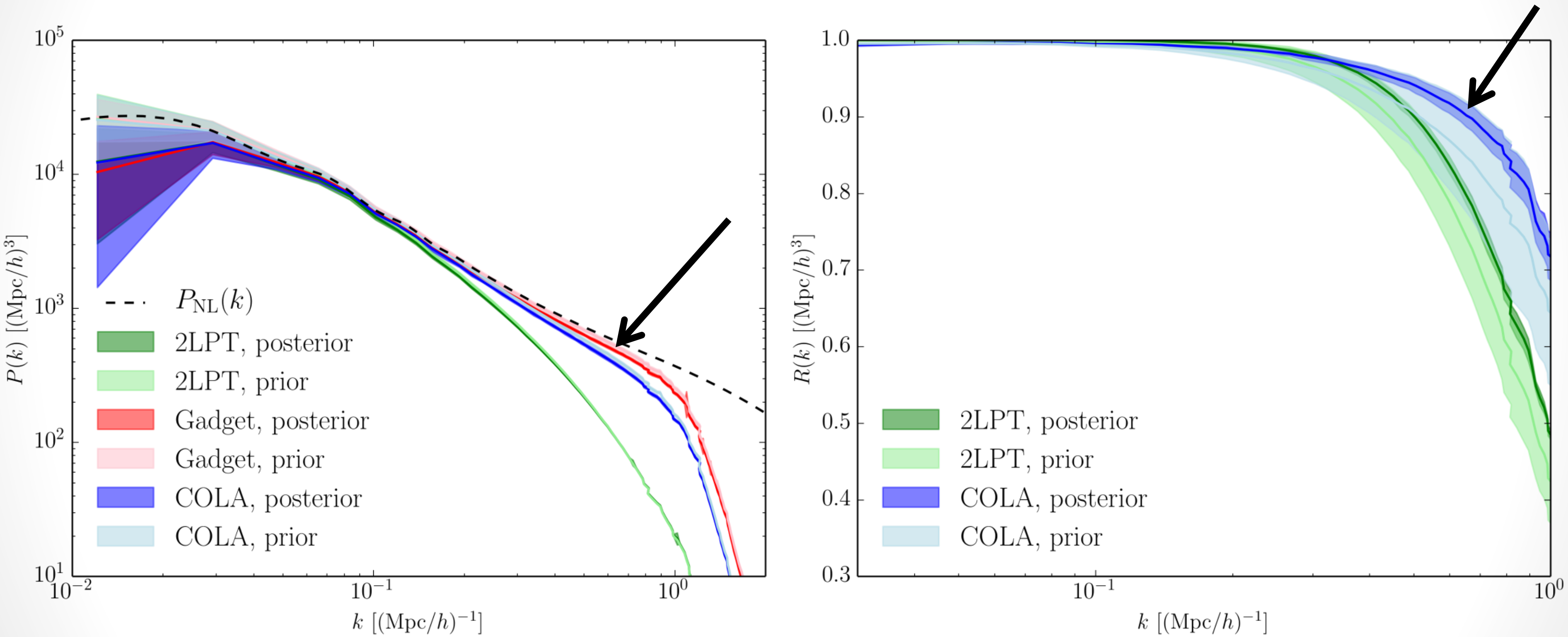
Modified:

$$\partial_\tau^2 \mathbf{s}_{\mathrm{MC}} = \partial_\tau^2(\mathbf{s} - \mathbf{s}_{\mathrm{LPT}}) = -\nabla\Phi - \partial_\tau^2 \mathbf{s}_{\mathrm{LPT}}$$



Tassev, Zaldarriaga & Einsenstein 2013, arXiv:1301.0322

Non-linear filtering improves the fit



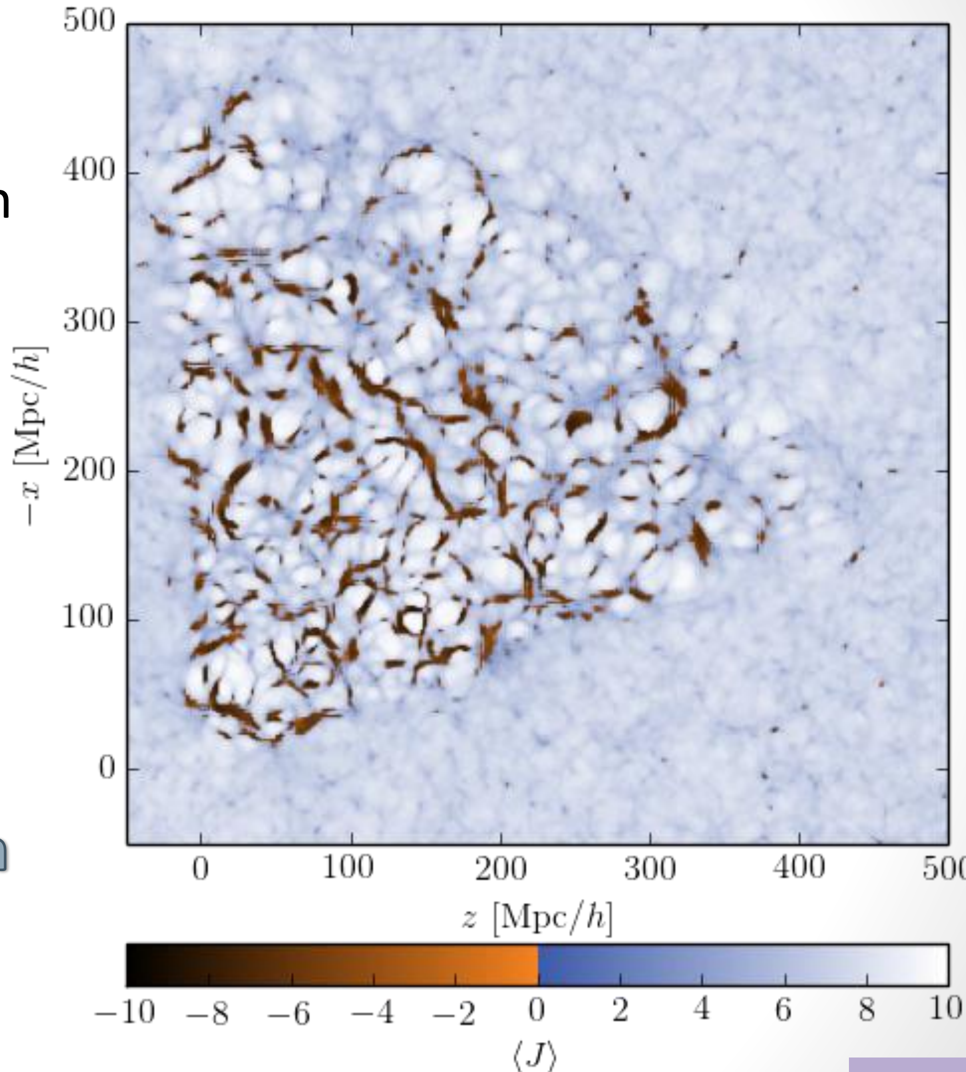
FL, arXiv:1512.04985 (chapter 7)

Inference of the dark matter phase-space sheet

- The dark matter phase-space sheet has been studied so far in simulations

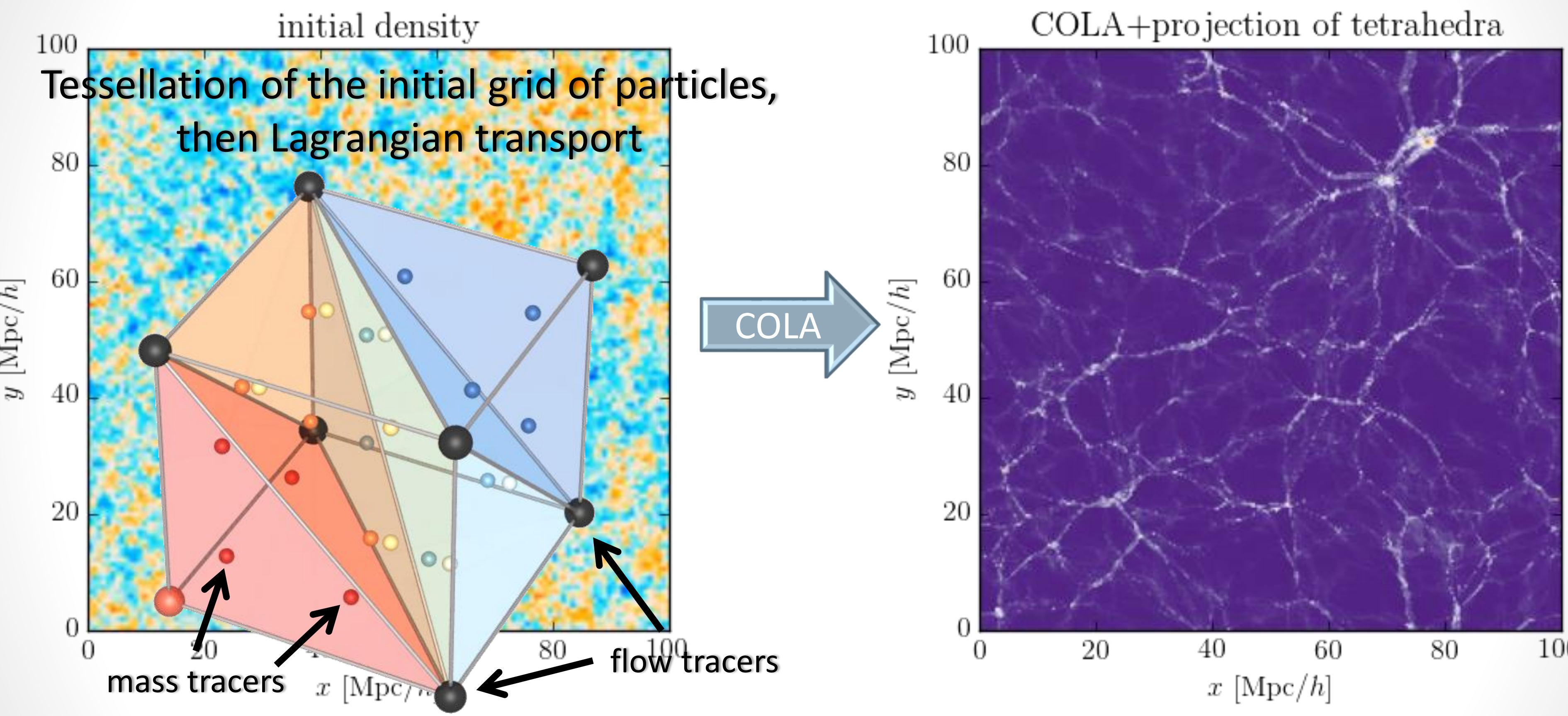
e.g. Neyrinck 2012, arXiv:1202.3364
Abel, Hahn & Kaehler 2012, arXiv:1111.3944
Shandarin, Habib & Heitmann 2012, arXiv:1111.2366

- BORG infers **Lagrangian dynamics** in real data
- Identified structures have a direct **physical interpretation**



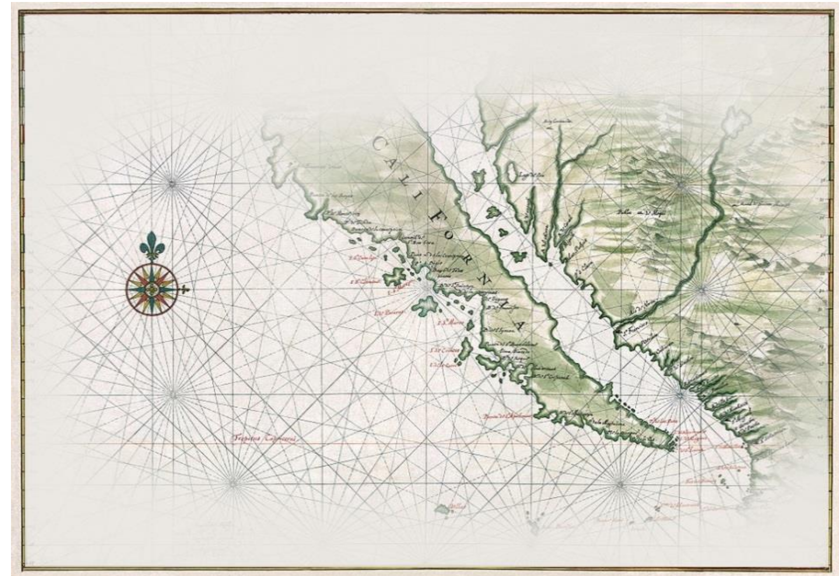
FL, Jasche, Lavaux & Wandelt 2016, arXiv:1601.00093

Non-linear filtering improves density samples

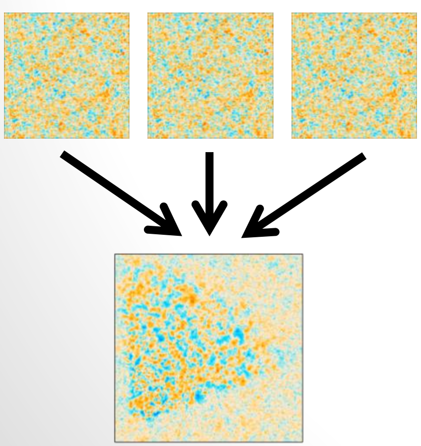


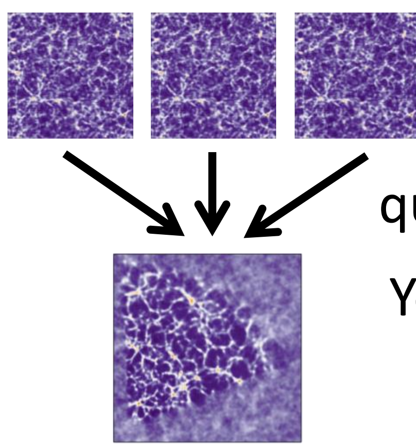
Hahn, Abel & Khaeler, arXiv:1210.6652

Uncertainty quantification



Uncertainty quantification is crucial!





Can we **propagate** uncertainty quantification to **cosmic web analysis**?

Yes, and this is what yields a connection with **information theory**!

Cosmic web classification procedures

void, sheet, filament, cluster?

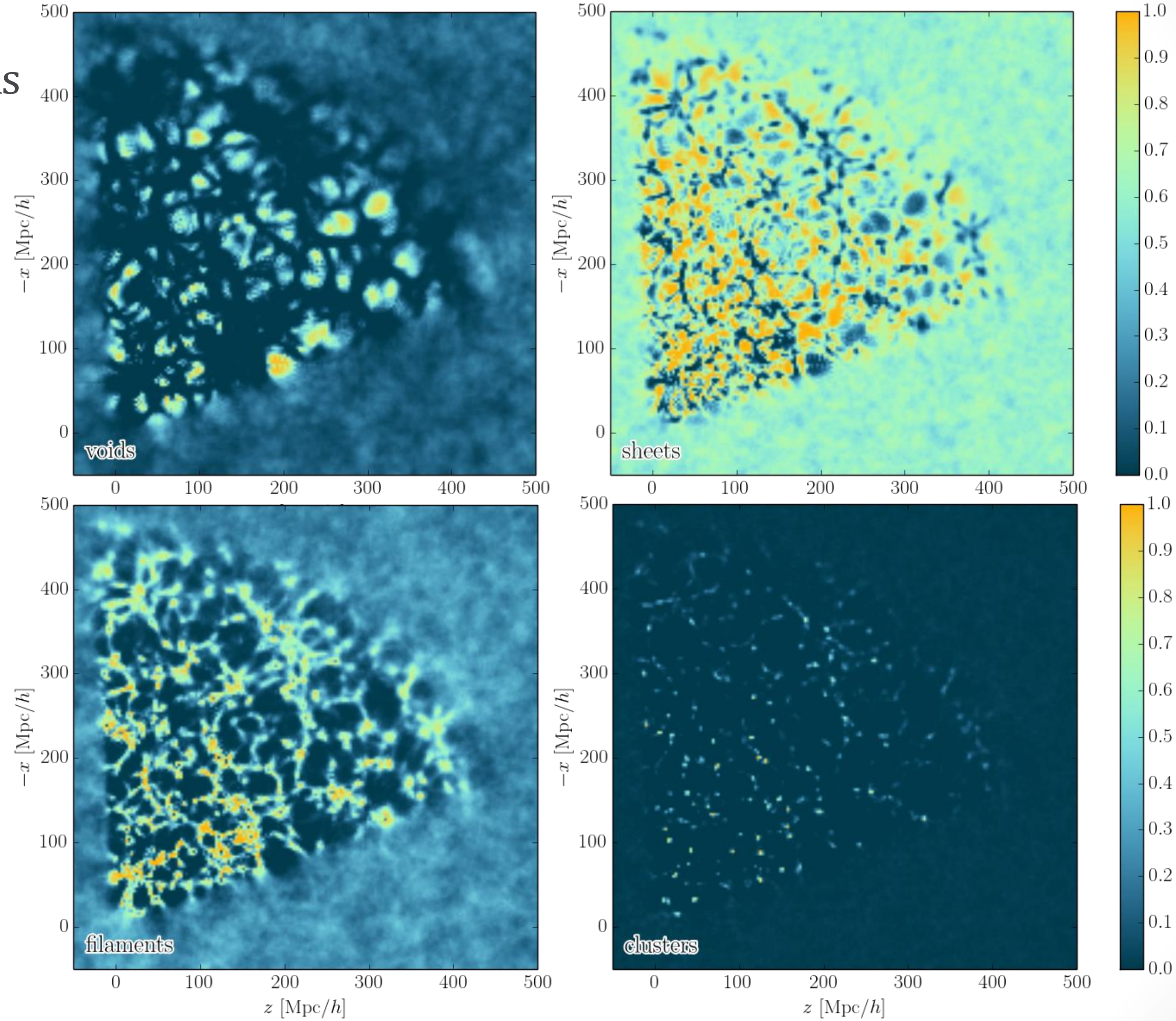
- The **T-web**:

uses the sign of μ_1, μ_2, μ_3: eigenvalues of the tidal field tensor, Hessian of the gravitational potential: $T_{ij}(\mathbf{x}) = \partial_i \partial_j \Phi(\mathbf{x})$

Hahn *et al.* 2007, arXiv:astro-ph/0610280

T-web structures inferred by BORG

Final conditions



FL, Jasche & Wandelt 2015, arXiv:1502.02690

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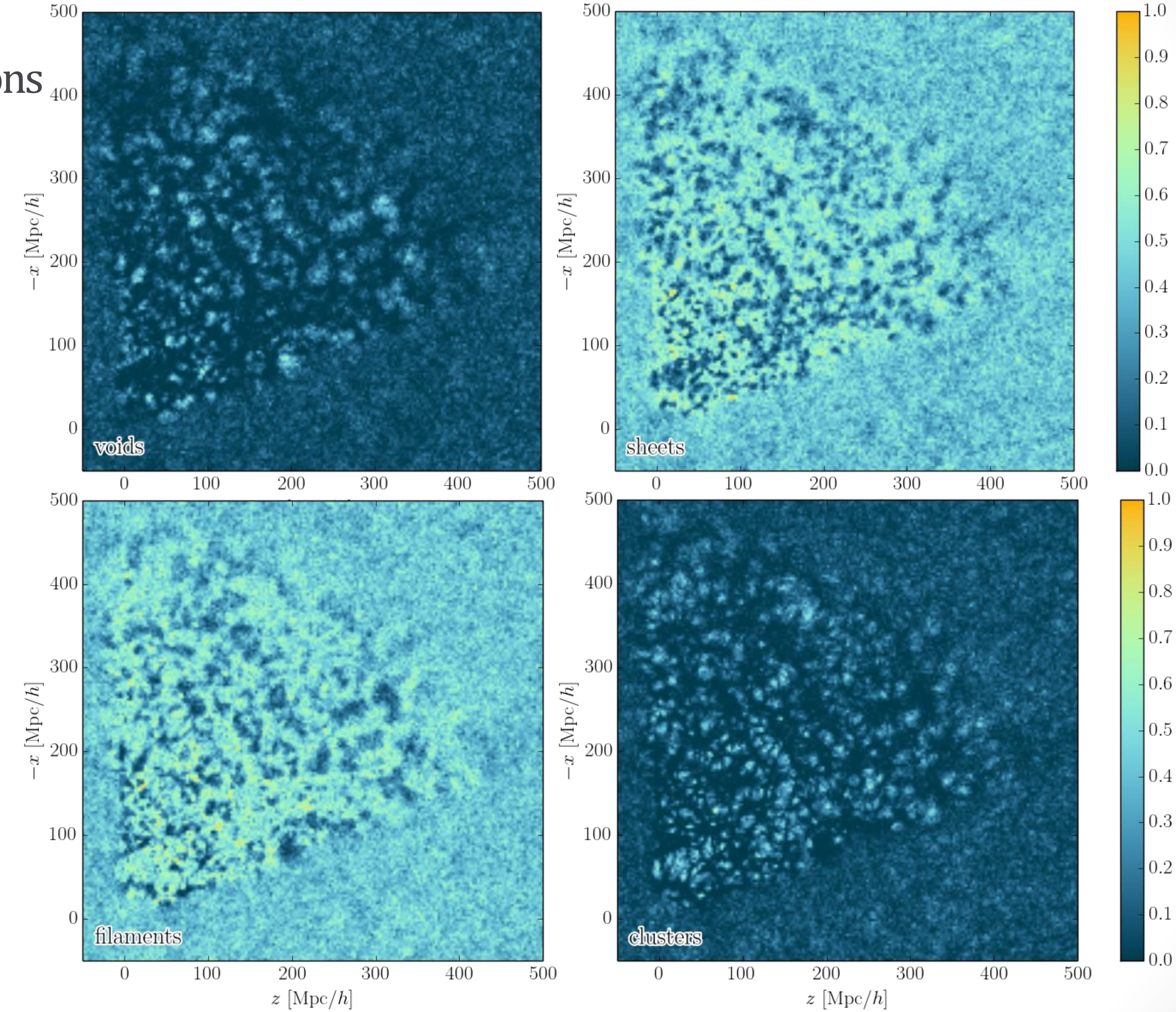
Initial conditions



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A decision rule for structure classification

- Space of "input features":

$$\{\mathrm{T}_0 = \text{void}, \mathrm{T}_1 = \text{sheet}, \mathrm{T}_2 = \text{filament}, \mathrm{T}_3 = \text{cluster}\}$$

- Space of "actions":

$$\{a_0 = \text{"decide void"}, a_1 = \text{"decide sheet"}, a_2 = \text{"decide filament"}, a_3 = \text{"decide cluster"}, a_{-1} = \text{"do not decide"}\}$$

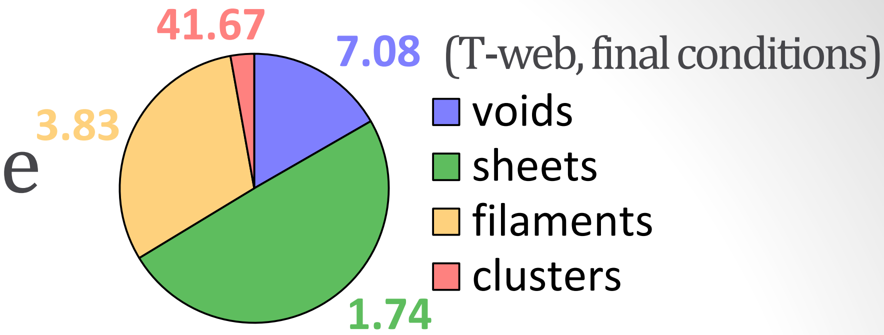
⇨ A problem of **Bayesian decision theory**:
one should take the action that maximizes the utility

$$U(a_j(\vec{x}_k)|d) = \sum_{i=0}^{3} G(a_j|\mathrm{T}_i)\,\mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d)$$

- How to write down the gain functions?

FL, Jasche & Wandelt 2015, arXiv:1503.00730

Gambling with the Universe



- One proposal:

$$G(a_j|\mathrm{T}_i) = \begin{cases} \dfrac{1}{\mathcal{P}(\mathrm{T}_i)} - \alpha & \text{if } j \in [\![0,3]\!] \text{ and } i = j \\ -\alpha & \text{if } j \in [\![0,3]\!] \text{ and } i \neq j \\ 0 & \text{if } j = -1. \end{cases}$$

"Winning"
"Loosing"
"Not playing"

- Without data, the expected utility is

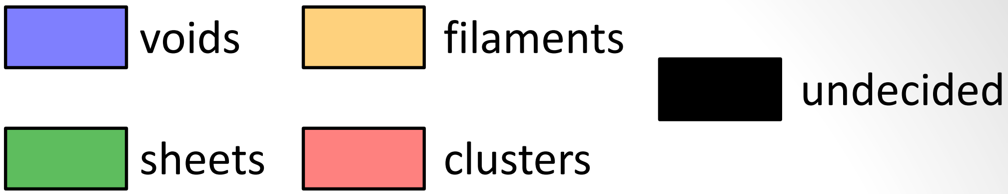
$$U(a_j) = 1 - \alpha \quad \text{if } j \neq 1$$ "Playing the game"

$$U(a_{-1}) = 0$$ "Not playing the game"

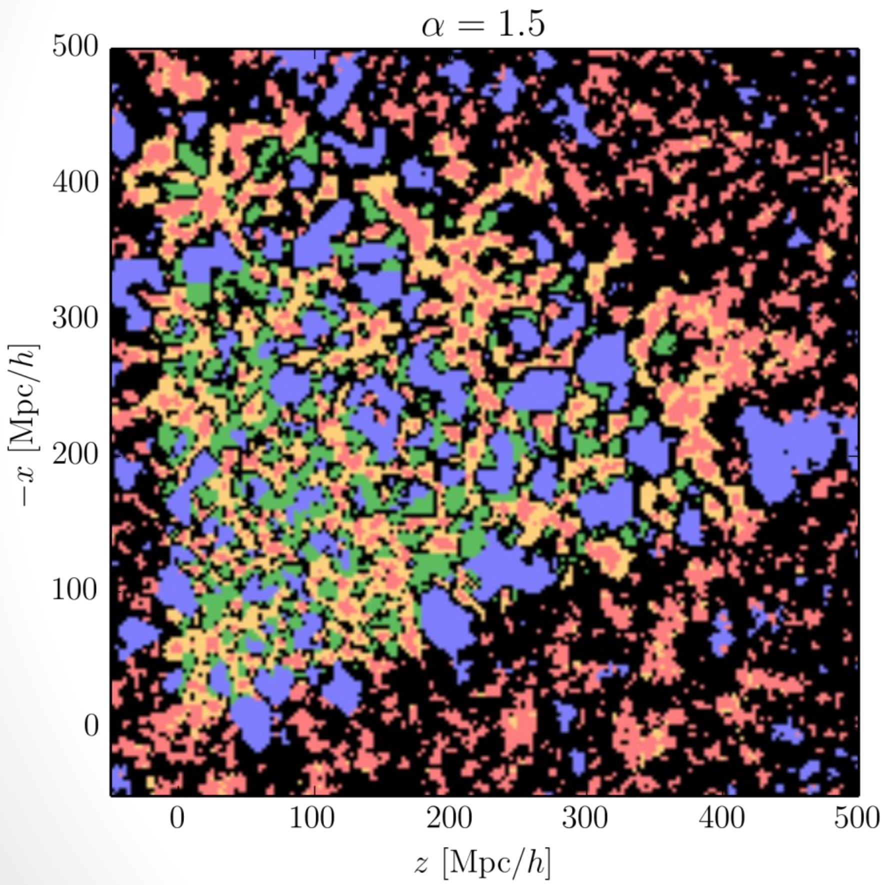
- With $\alpha = 1$, it's a *fair game* ⇨ always play
  ⇨ "speculative map" of the LSS
- Values $\alpha > 1$ represent an *aversion for risk*
  ⇨ increasingly "conservative maps" of the LSS

FL, Jasche & Wandelt 2015, arXiv:1503.00730

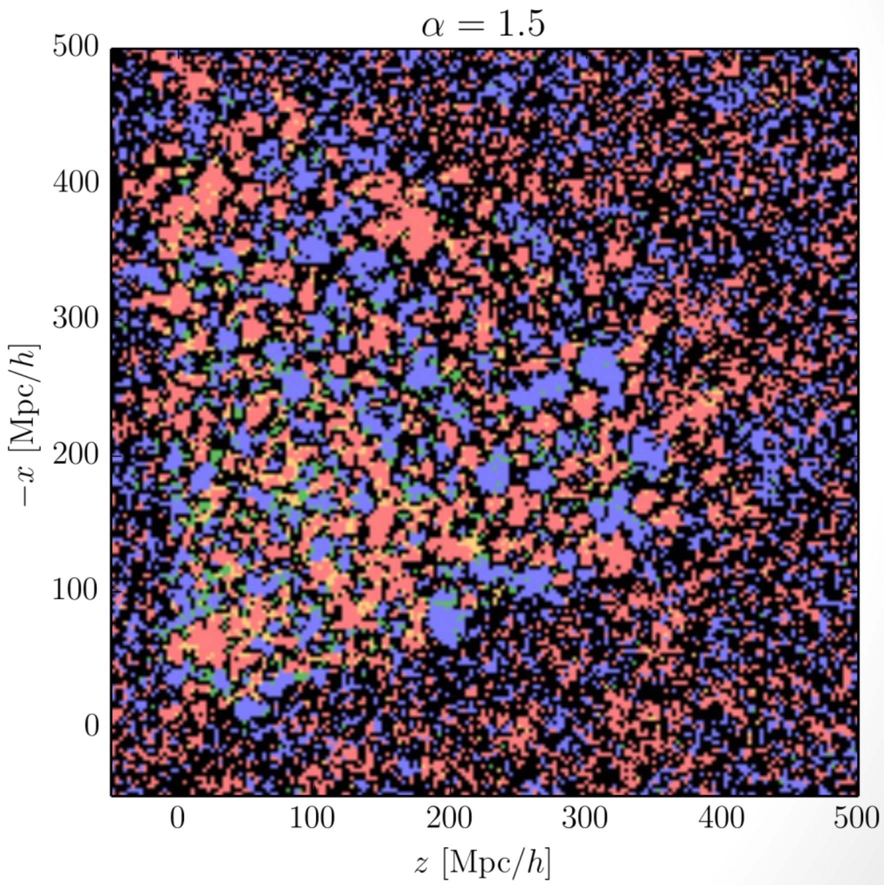
Playing the game…



Final conditions



Initial conditions



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Hahn *et al.* 2007, arXiv:astro-ph/0610280

- **DIVA**:

uses the sign of $\lambda_1, \lambda_2, \lambda_3$: eigenvalues of the shear of the Lagrangian displacement field: $R_{\ell m}(\mathbf{q}) = \partial_m \Psi_\ell(\mathbf{q})$

Lavaux & Wandelt 2010, arXiv:0906.4101

- **ORIGAMI** :

uses the dark matter "phase-space sheet" (number of orthogonal axes along which there is shell-crossing)

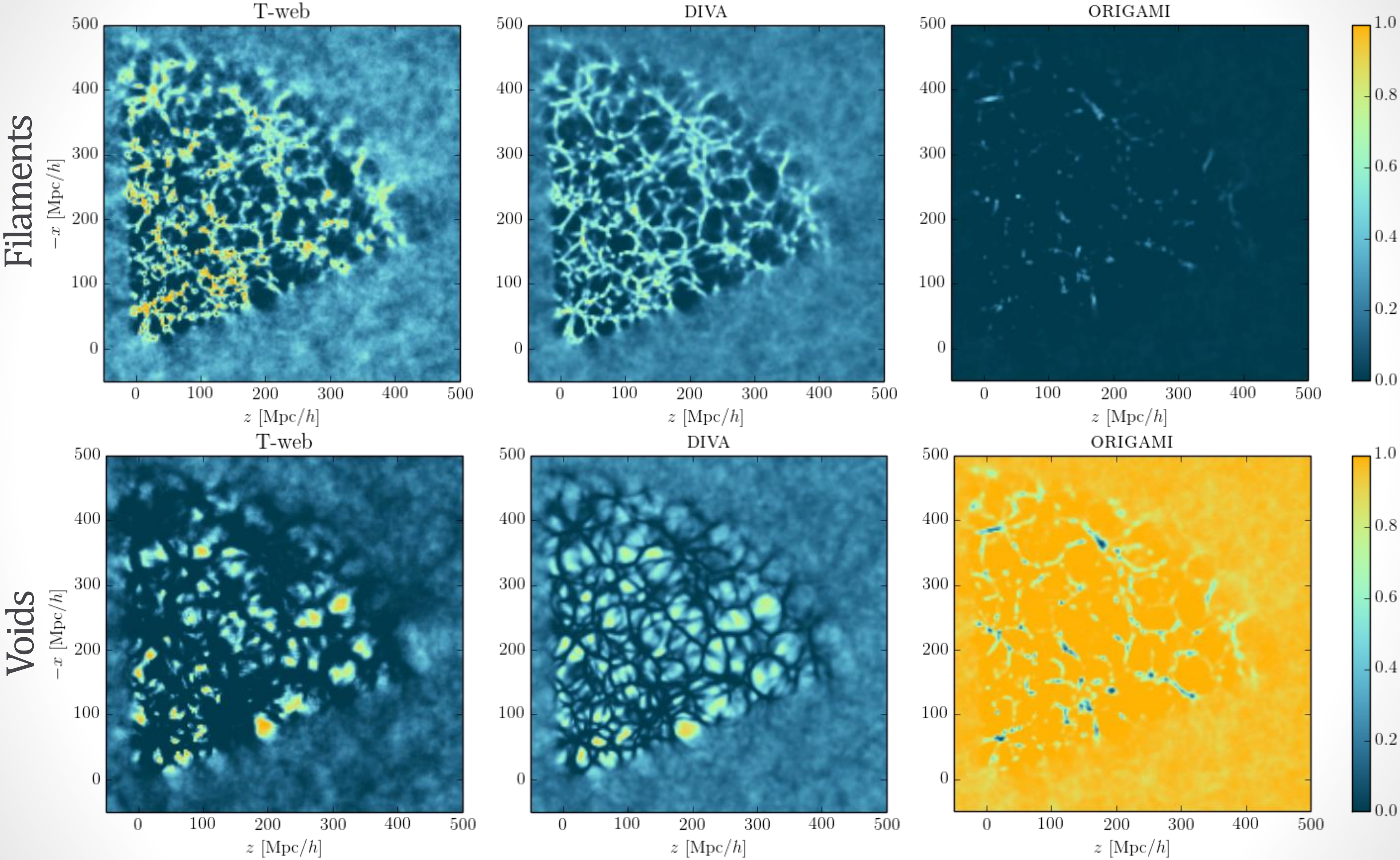
Falck, Neyrinck & Szalay 2012, arXiv:1201.2353

Lagrangian classifiers

now usable in real data!

and many others…

Comparing classifiers



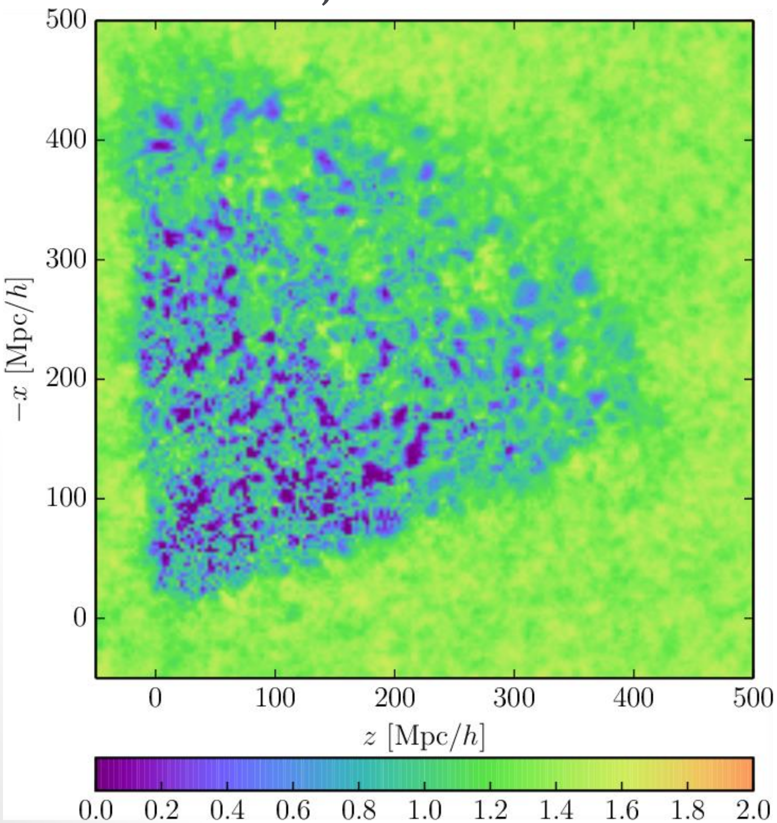
FL, Jasche & Wandelt 2015, arXiv:1502.02690
FL, Jasche, Lavaux & Wandelt 2016, arXiv:1601.00093

What is the information content of these maps?

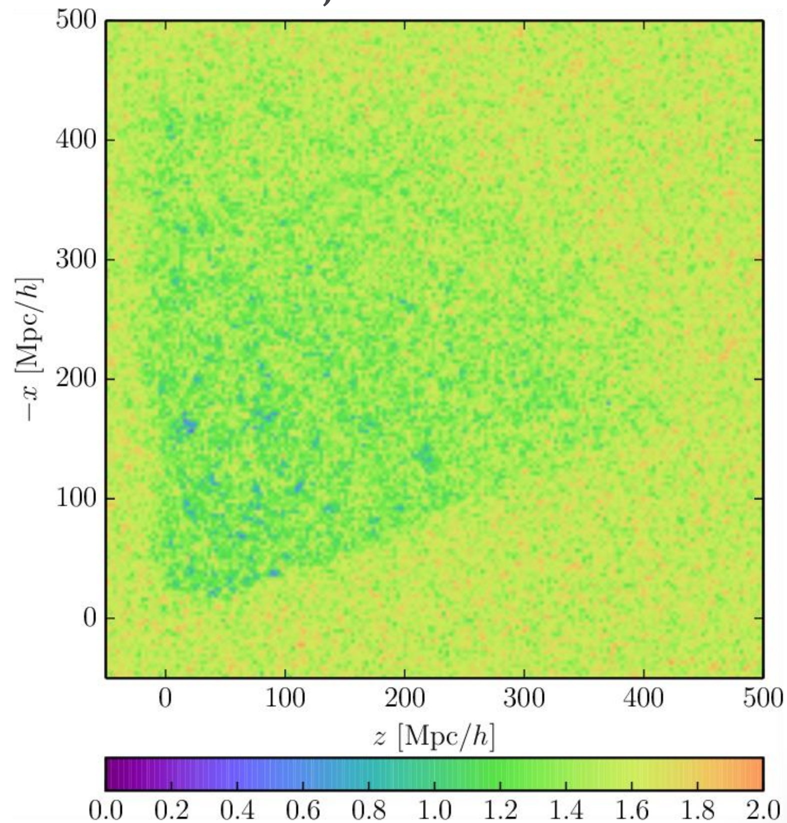
Shannon entropy

$$H\left[\mathcal{P}(\mathrm{T}(\vec{x}_k)|d)\right] \equiv -\sum_{i=0}^{3} \mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d)\log_2(\mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d))$$ in shannons (Sh)

T-web, final conditions



T-web, initial conditions

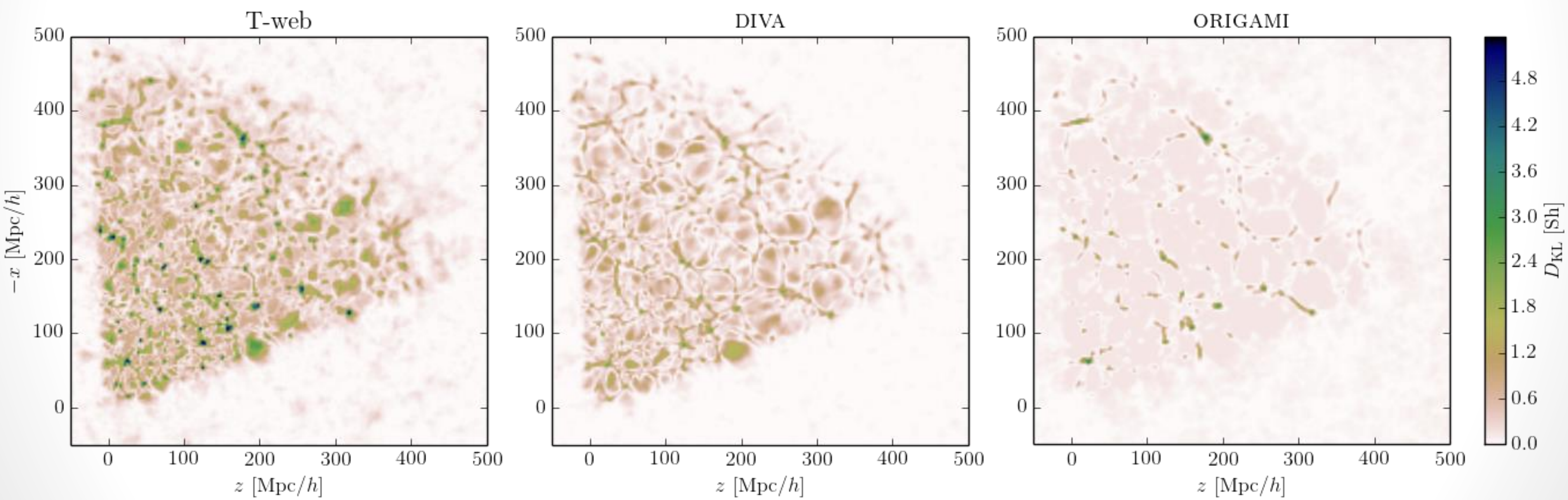


FL, Jasche & Wandelt 2015, arXiv:1502.02690

How much did the data surprise us?

information gain a.k.a. relative entropy or Kullback-Leibler divergence posterior/prior

$$D_{\mathrm{KL}}\left[\mathcal{P}(\mathrm{T}(\vec{x}_k)|d)||\mathcal{P}(\mathrm{T})\right] = \sum_i \mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d)\log_2\left(\frac{\mathcal{P}(\mathrm{T}_i(\vec{x}_k)|d)}{\mathcal{P}(\mathrm{T}_i)}\right) \quad \text{in Sh}$$



(more about the Kullback-Leibler divergence later)

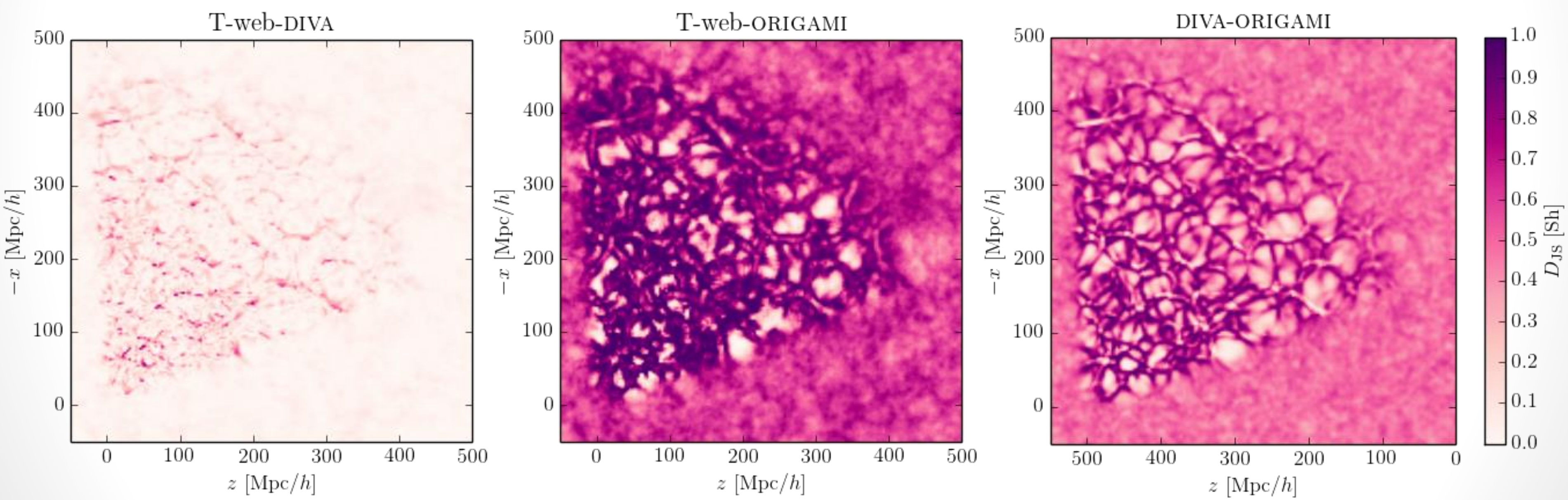
FL, Jasche & Wandelt 2015, arXiv:1502.02690

How similar are different classifications?

Jensen-Shannon divergence

$$D_{\mathrm{JS}}[\mathcal{P}:\mathcal{Q}] \equiv \frac{1}{2}D_{\mathrm{KL}}\left[\mathcal{P}||\frac{\mathcal{P}+\mathcal{Q}}{2}\right] + \frac{1}{2}D_{\mathrm{KL}}\left[\mathcal{Q}||\frac{\mathcal{P}+\mathcal{Q}}{2}\right]$$

in Sh,
between 0 and 1



(more about the Jensen-Shannon divergence later)

FL, Lavaux, Jasche & Wandelt 2016, in prep.

Which is the best classifier?

- **Decision theory**: a framework to classify structures in the presence of uncertainty.
  Can we extend the decision problem to the space of classifiers?
- As before, the idea is to maximize a utility function

$$U(\xi) = \langle U(d, \mathrm{T}, \xi) \rangle_{\mathcal{P}(d,\mathrm{T}|\xi)}$$

- An important notion: the **mutual information** between two random variables

$$I\left[X:Y\right] \equiv D_{\mathrm{KL}}[\mathcal{P}(x,y)||\mathcal{P}(x)\mathcal{P}(y)]$$
$$= \sum_{x \in \mathcal{X}, y \in \mathcal{Y}} \mathcal{P}(x,y) \log_2 \left(\frac{\mathcal{P}(x,y)}{\mathcal{P}(x)\mathcal{P}(y)} \right)$$

- Property: $$I\left[X:Y\right] = \langle D_{\mathrm{KL}}[\mathcal{P}(x|y)||\mathcal{P}(x)] \rangle_{\mathcal{P}(Y)}$$

Mutual information is the expectation of the Kullback-Leibler divergence of the conditional from the unconditional distribution.

FL, Lavaux, Jasche & Wandelt 2016, in prep.

1. Utility for parameter inference:

example: cosmic web analysis

- **Example:** *Which classifier produces the most "surprising" cosmic web maps when looking at the data?*

- In analogy with the formalism of **Bayesian experimental design**: maximize the **expected information gain** for cosmic web maps

$$U_1(d, \xi)(\vec{x}_k) = D_{\mathrm{KL}}\left[\mathcal{P}(\mathrm{T}(\vec{x}_k)|d, \xi)||\mathcal{P}(\mathrm{T}|\xi)\right]$$

$$U_1(\xi) = I[\mathrm{T} : d|\xi]$$

classification (T) data (d)

FL, Lavaux, Jasche & Wandelt 2016, in prep.

2. Utility for model selection:
example: dark energy equation of state

- **Example:** *Let us consider three dark energy models with*

$$w = -0.9, w = -1, w = -1.1 \ .$$

Which classifier separates them better?

- The **Jensen-Shannon divergence** between posterior predictive distributions can be used as an approximate **predictor for the change in the Bayes factor**

Vanlier *et al.* 2014, BMC Syst Biol 8, 20 (2014)

- In analogy: $U_2(d,\xi)(\vec{x}_k) = D_{\mathrm{JS}}\left[\mathcal{P}(\mathrm{T}(\vec{x}_k)|d,\mathcal{M}_1) : \mathcal{P}(\mathrm{T}(\vec{x}_k)|d,\mathcal{M}_2)|\xi\right]$

$$U_2(\xi) = I\left[\mathcal{M} : \mathcal{R}(d)|\xi\right]$$

model classifier ($\mathcal{M}$) mixture distribution ($\mathcal{R}(d)$)

$$\mathcal{R}(d) \equiv \frac{\mathcal{P}(\mathrm{T}(\vec{x}_k)|d,\mathcal{M}_1) + \mathcal{P}(\mathrm{T}(\vec{x}_k)|d,\mathcal{M}_2)}{2}$$

FL, Lavaux, Jasche & Wandelt 2016, in prep.

3. Utility for prediction of new data:
example: galaxy colors

- **Example:** *So far we have not used galaxy colors. Which classifier predicts them best?*

- Maximize the **expected information gain** for some new quantity

$$U_3(d, \mathrm{T}, \xi) = D_{\mathrm{KL}} \left[\mathcal{P}(c|d, \mathrm{T}, \xi) || \mathcal{P}(c|\xi) \right]$$

$$U_3(\xi) = I[c : \mathrm{T}|\xi]$$

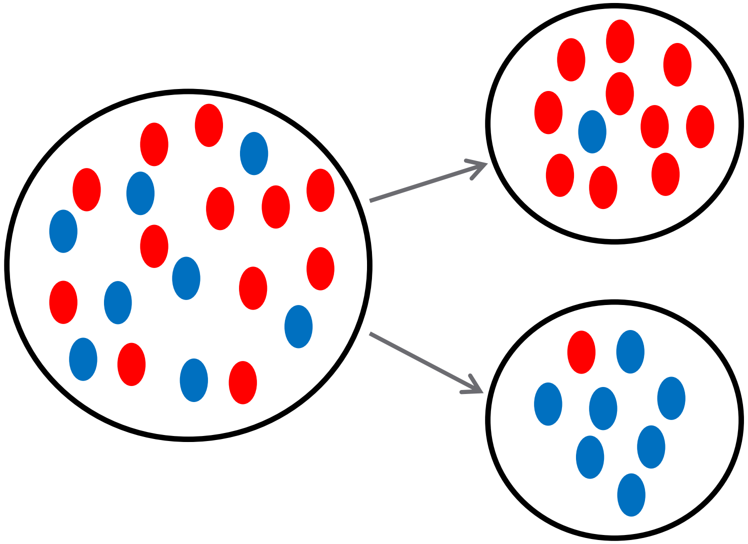
predicted data (c) — classification (T)

FL, Lavaux, Jasche & Wandelt 2016, in prep.

3. Utility for prediction of new data:

example: galaxy colors

- How to compute the information gain?



child1 entropy:

$$H = -\frac{10}{11}\log_2\left(\frac{10}{11}\right) - \frac{1}{11}\log_2\left(\frac{1}{11}\right) = 0.4395$$

child2 entropy:

$$H = -\frac{8}{9}\log_2\left(\frac{8}{9}\right) - \frac{1}{9}\log_2\left(\frac{1}{9}\right) = 0.5033$$

parent entropy:

$$H = -\frac{8}{20}\log_2\left(\frac{8}{20}\right) - \frac{12}{20}\log_2\left(\frac{12}{20}\right) = 0.9709$$

weighted average entropy of children:

$$\frac{11}{20} \times 0.4395 + \frac{9}{20} \times 0.5033 = 0.4682$$

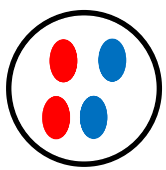
information gain for this split: $0.9709 - 0.4682 = 0.5027 \text{ Sh}$

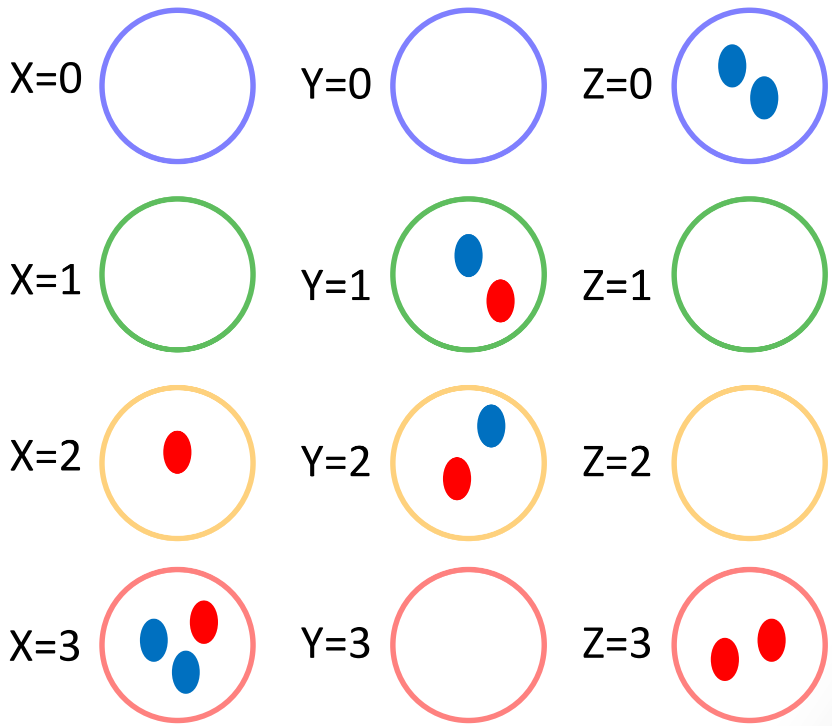
3. Utility for prediction of new data:

example: galaxy colors

- A **supervised machine learning** problem!
  - 3 **features** = classifications (T-web, DIVA, ORIGAMI) with
  - 4 **possible values** (void, sheet, filament, cluster)
  - 2 **classes** (red, blue)

X	Y	Z	C
3	2	3	I
3	1	3	I
2	2	0	II
3	1	0	II





$\mathrm{IG} = 0.3112$ $\mathrm{IG} = 0.$ no gain: worst $\mathrm{IG} = 1.$ best!

FL, Lavaux, Jasche & Wandelt 2016, in prep.

Summary & Conclusions

- Thanks to **BORG**, the **cosmic web** can be described using various classifiers.
- Probabilistic analysis of the cosmic web yields a data-supported **connection between cosmology and information theory**.
- **Decision theory** offers a framework to classify structures in the presence of uncertainty.
- The decision problem can be extended to the **space of classifiers**, with utility functions depending on the desired use.

(Some numerical results for classifier utilities in the upcoming paper)

All maps, catalogs & scripts are publicly available at http://icg.port.ac.uk/~leclercq/

References

Jasche & Wandelt 2013, arXiv:1203.3639	(BORG proof of concept)
Jasche, FL & Wandelt 2015, arXiv:1409.6308	(BORG SDSS analysis)
FL, Jasche & Wandelt 2015, arXiv:1502.02690	(T-web, entropy, relative entropy)
FL, Jasche & Wandelt 2015, arXiv:1503.00730	(decision theory)
FL, Jasche, Lavaux & Wandelt 2016, arXiv:1601.00093	(DIVA & ORIGAMI)
FL, Lavaux, Jasche & Wandelt 2016, in prep. (very soon)	(mutual information, classifier utilities)