



Cosmic web analysis and information theory

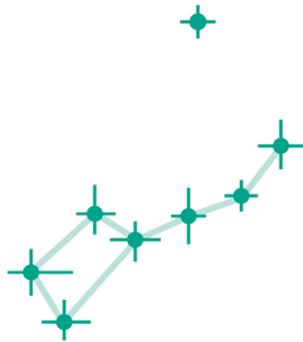
Florent Leclercq

www.florent-leclercq.eu

Imperial Centre for Inference and Cosmology
Imperial College London

Guilhem Lavaux, Jens Jasche, Alan Heavens,
and the Aquila Consortium
www.aquila-consortium.org

June 17th, 2019



ICIC

Imperial Centre
for Inference & Cosmology

**Imperial College
London**

The BORG inference framework

Bayesian Origin Reconstruction from Galaxies

- **A Bayesian Hierarchical Model:**

$$\mathcal{P}(\hat{\delta}) \propto \exp\left(-\frac{1}{2} \sum_k \frac{|\hat{\delta}_k|^2}{P_k}\right) \quad \text{initial conditions}$$

$$\rho_m = \mathcal{F}(\delta) \quad \text{total evolved matter density}$$

$$\rho_g = \mathcal{B}(\rho_m) \quad \text{biased galaxy distribution}$$

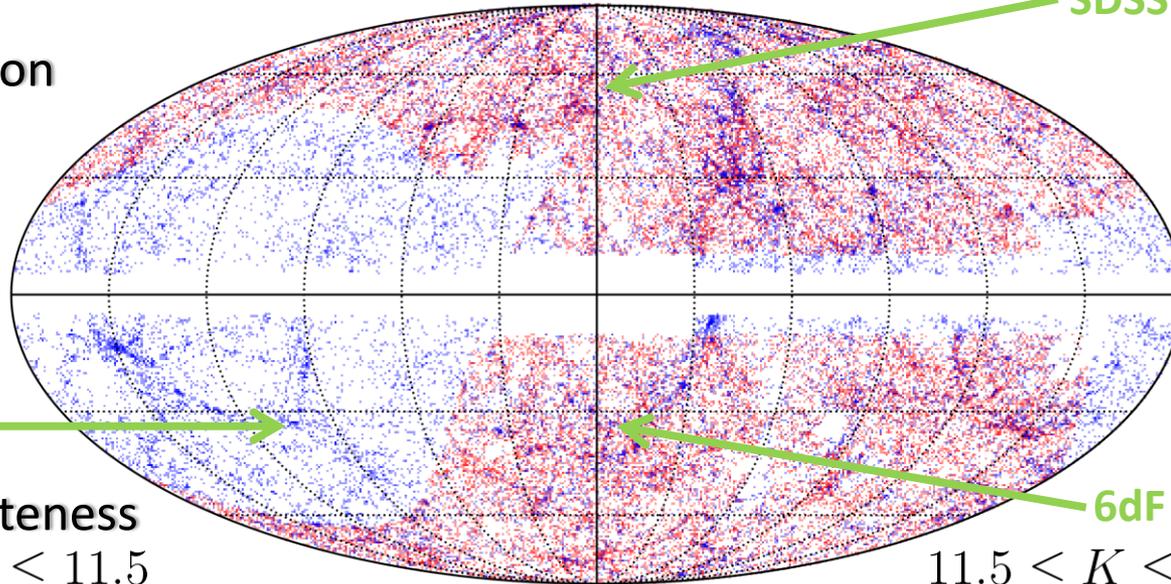
$$\rho_g^s(\vec{x}) = S(\vec{x})\rho_g(\vec{x}) \quad \text{selected sample}$$

$$N_g \curvearrowright \mathcal{P}(N_g | \rho_g^s) \quad \text{galaxy number count:} \\ \text{random extraction (Poisson,} \\ \text{Negative Binomial)}$$

- The multi-million dimensional posterior distribution is sampled via **Hamiltonian Monte Carlo**.

The 2M++ galaxy compilation

Galaxy distribution



SDSS

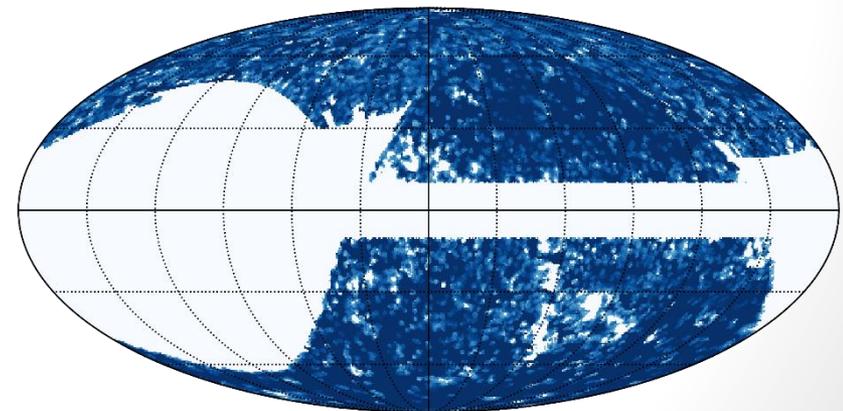
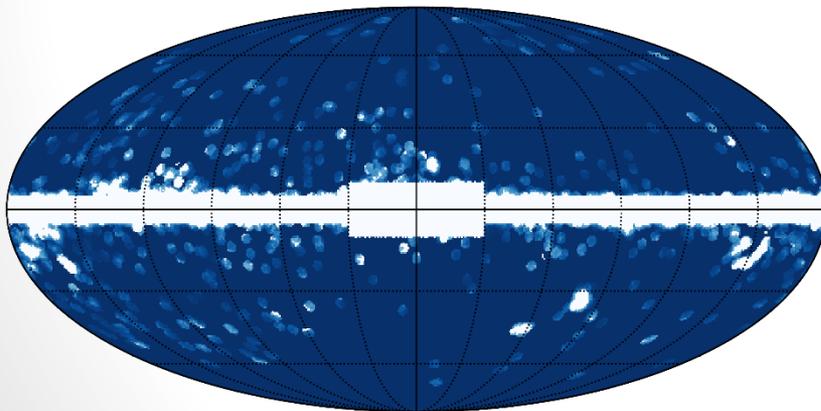
2MRS

6dF

Redshift completeness

$$K \leq 11.5$$

$$11.5 \leq K \leq 12.5$$



Lavaux & Hudson 2011, 1105.6107

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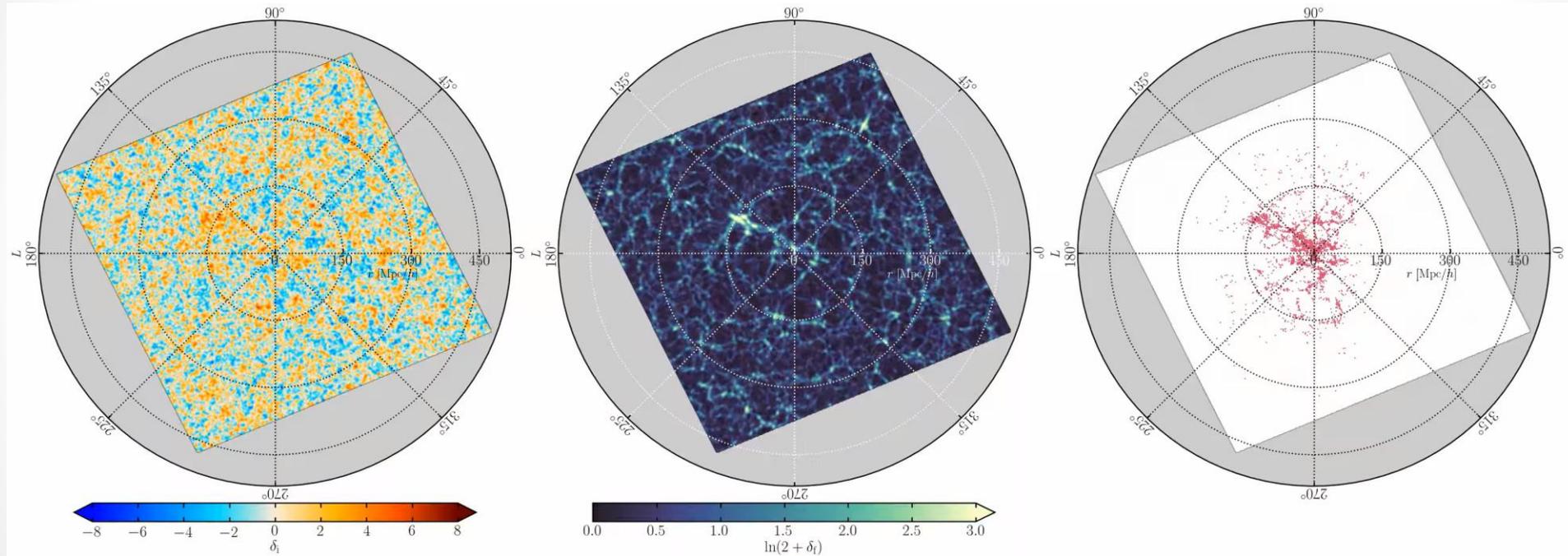
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BORG at work: Bayesian chrono-cosmography

Initial conditions

Final conditions

Observations



Supergalactic plane

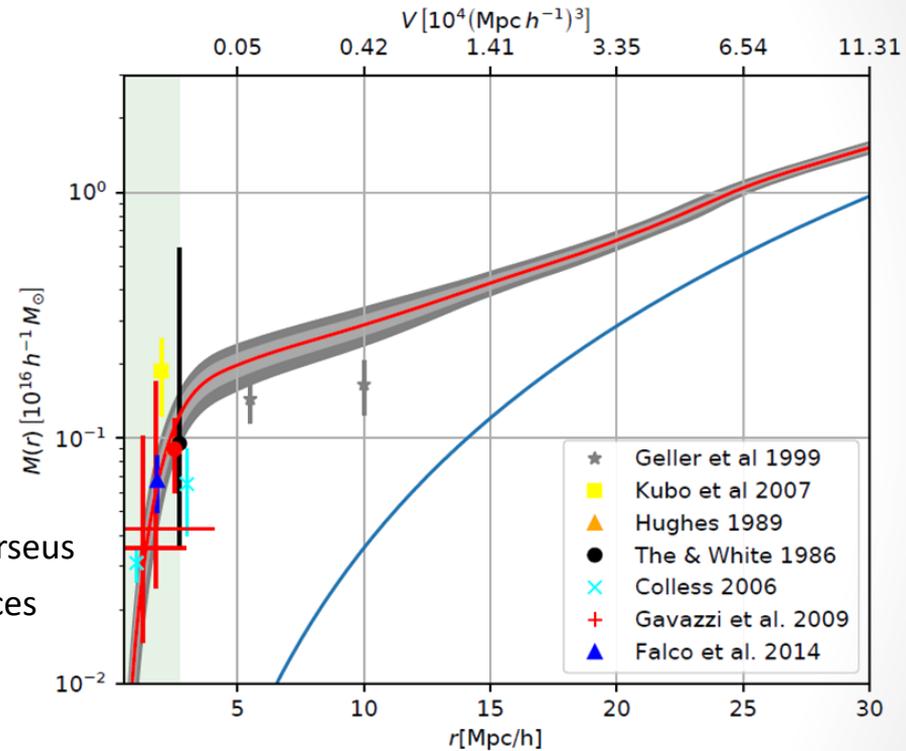
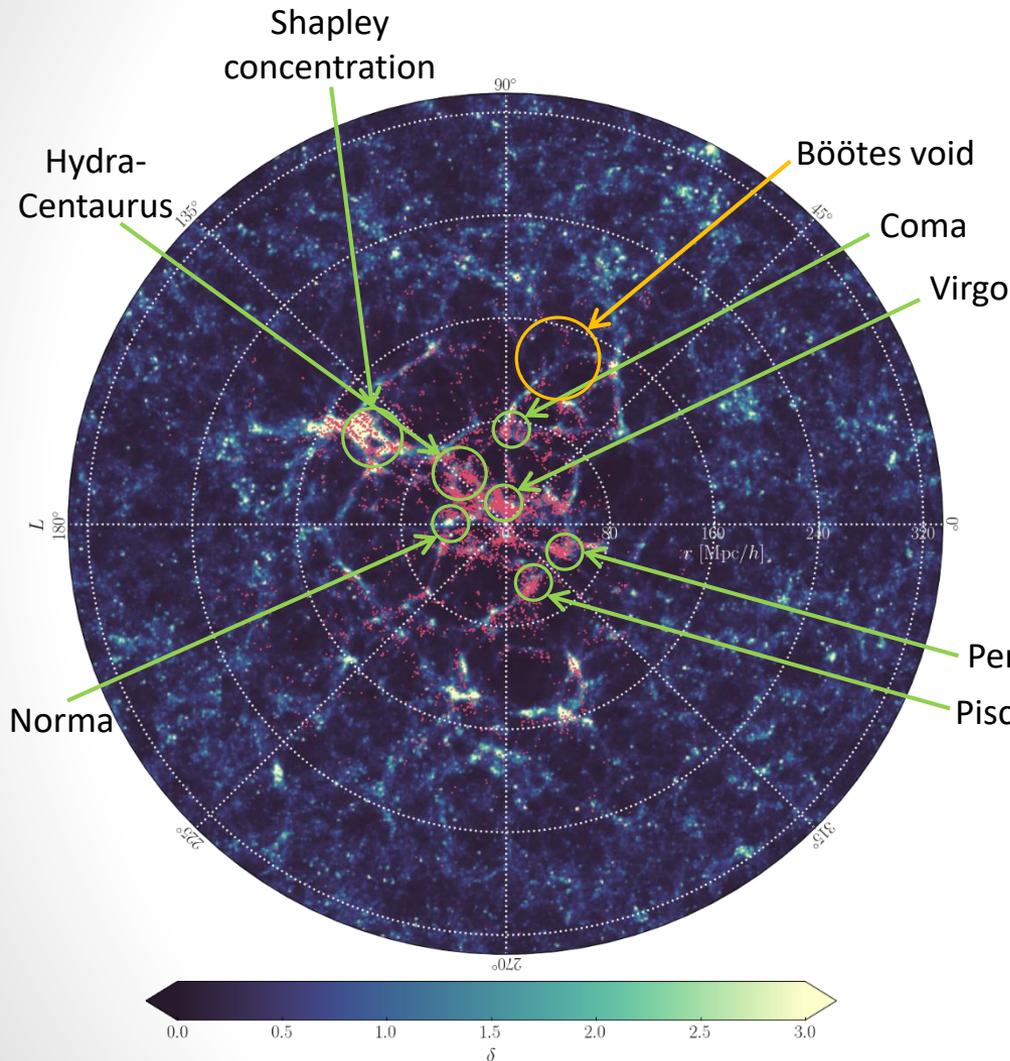
67,224 galaxies, \approx 17 million parameters, 5 TB of primary data products, 10,000 samples, \approx 500,000 forward and adjoint data model evaluations, 1.5 million CPU-hours

Jasche & Lavaux 2019, 1806.11117

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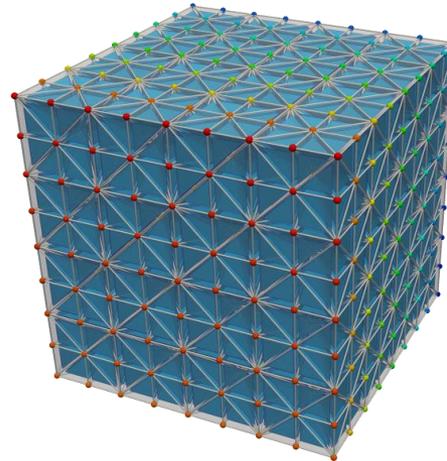
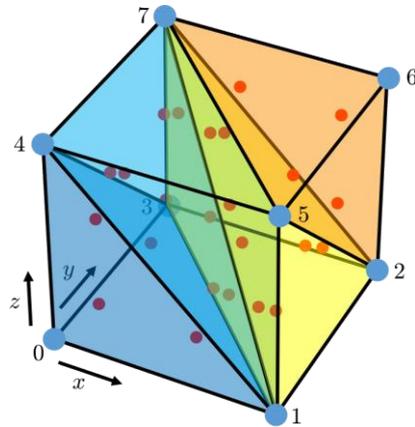
BORGPM density field: full non-linear dynamics



Mass profile of the **Coma cluster**, in agreement with gravitational lensing and X-ray observations down to a few Mpc.

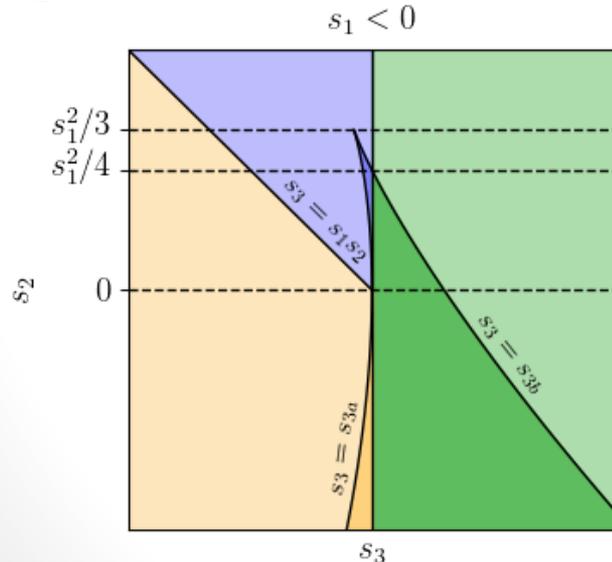
The phase-space structure of dark matter: tools

Delaunay tessellation of elementary Lagrangian cubes



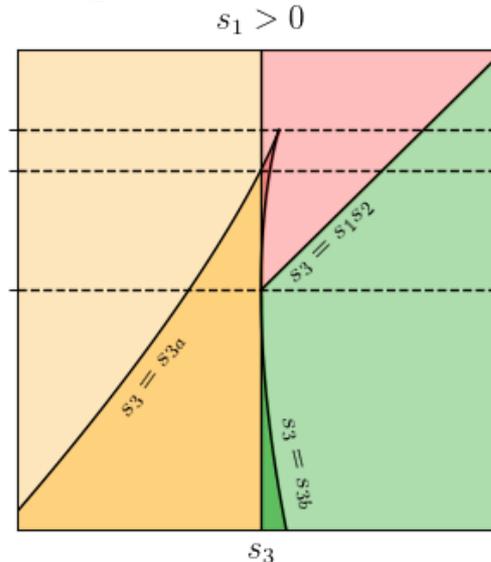
Abel, Hahn & Kaehler 2012, 1111.3944
 Shandarin, Habib & Heitmann 2012, 1111.2366
 Hahn, Abel & Kaehler 2013, 1210.6652
 Hahn & Angulo 2016, 1501.01959
 Sousbie & Colombi 2016, 1509.07720

Lagrangian Invariants Classification of Heterogeneous flows (LICH)



FL, Jasche, Lavaux, Wandelt & Percival 2017, 1601.00093

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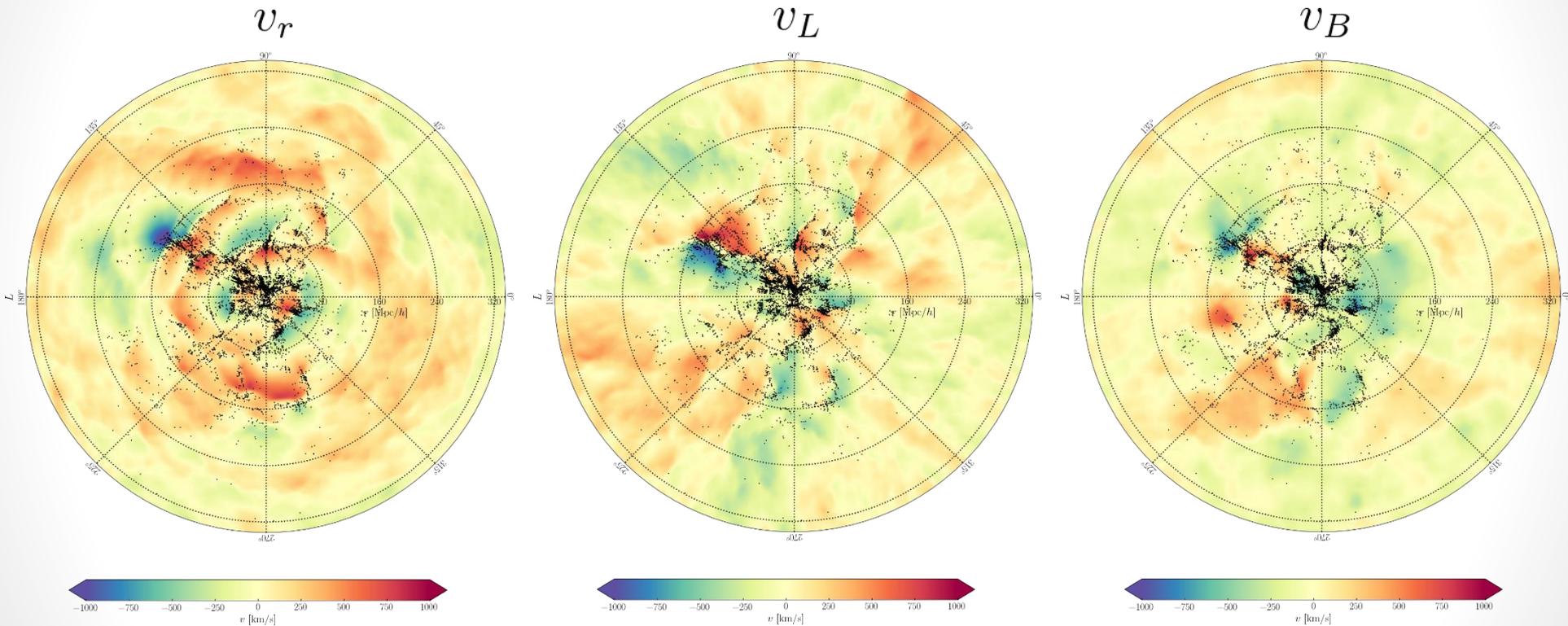
$$\mathcal{R}_{\ell m} \equiv \frac{\partial \Psi_\ell}{\partial \mathbf{q}_m}$$

$$\lambda^3 + \boxed{s_1} \lambda^2 + \boxed{s_2} \lambda + \boxed{s_3} = 0$$

- potential clusters
- vortical clusters
- potential filaments
- vortical filaments
- potential sheets
- vortical sheets
- potential voids
- vortical voids

Generalises DIVA,
 Lavaux & Wandelt 2010, 0906.4101

Velocity field in the supergalactic plane



The **gravitational infall** of known structures can be observed.

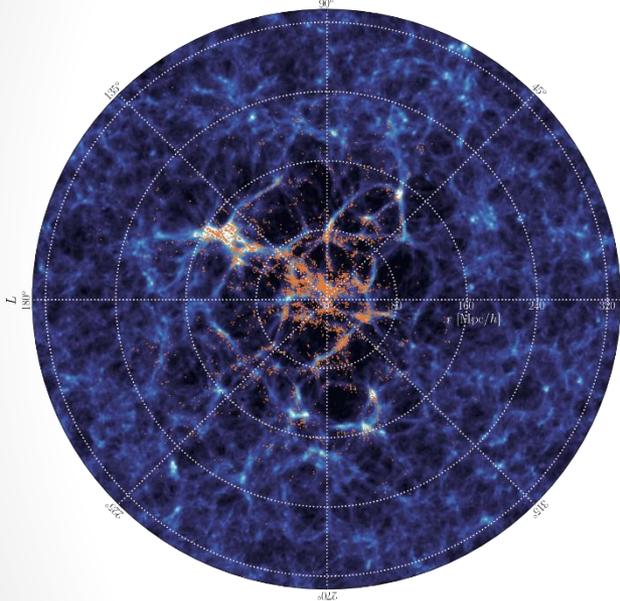
FL, Lavaux & Jasche, in prep.

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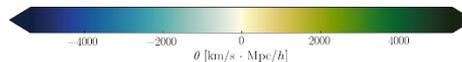
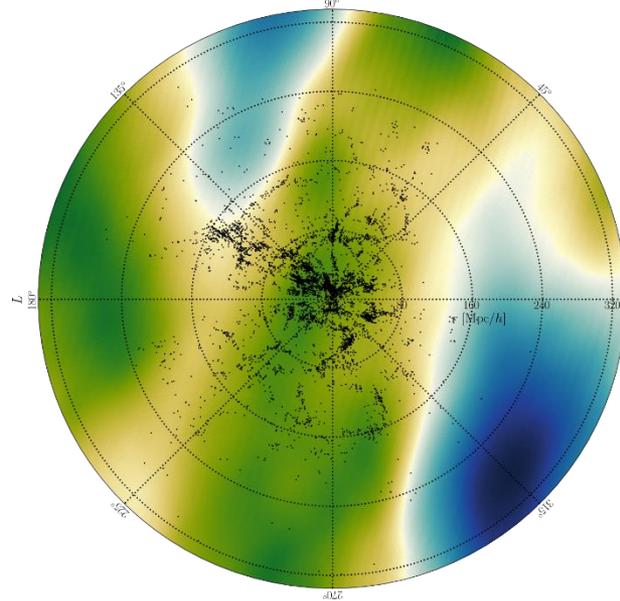
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Number of streams and vorticity

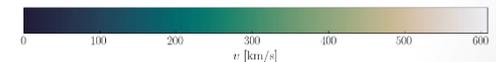
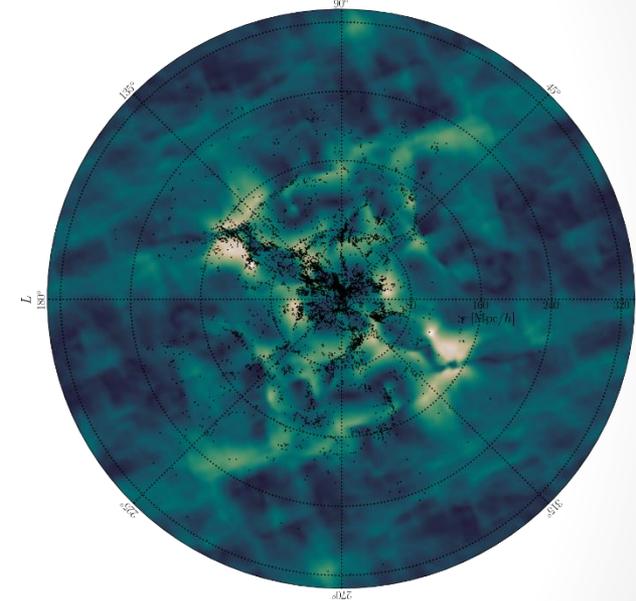
Number of streams



Velocity potential



Norm of vorticity

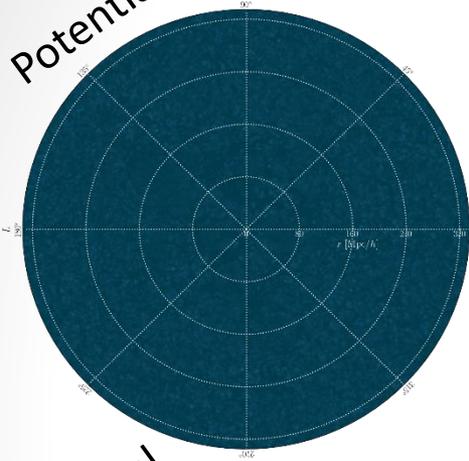


In earlier work (Leclercq, Jasche, Lavaux, Wandelt & Percival 2017, arXiv:1601.00093), these were postdictions. Thanks to BORGPM (full non-linear dynamics), we have now actual measurements - with uncertainties.

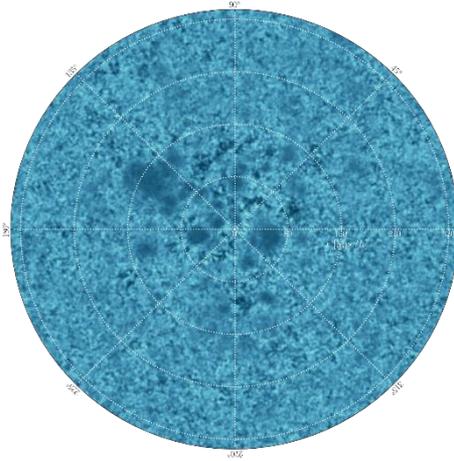
FL, Lavaux & Jasche, in prep.

LICH initial structures inferred by BORG

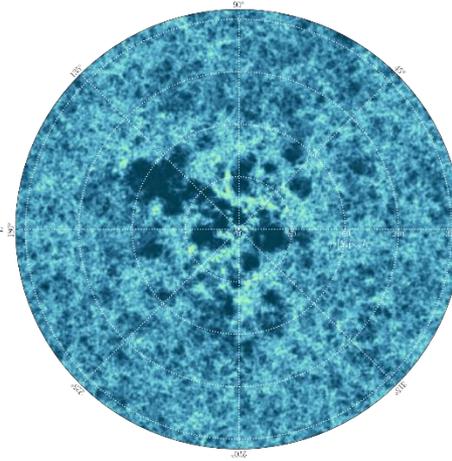
Potential Clusters



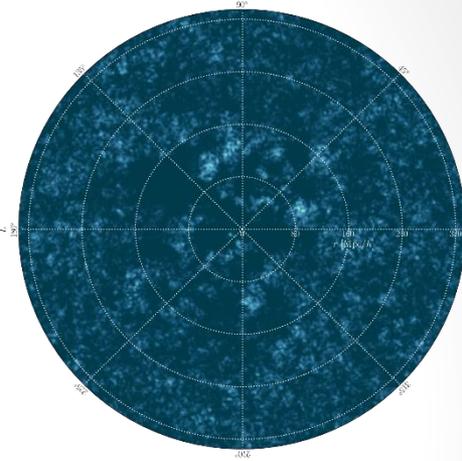
Filaments



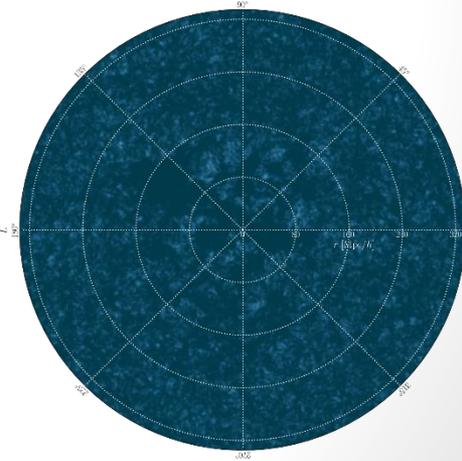
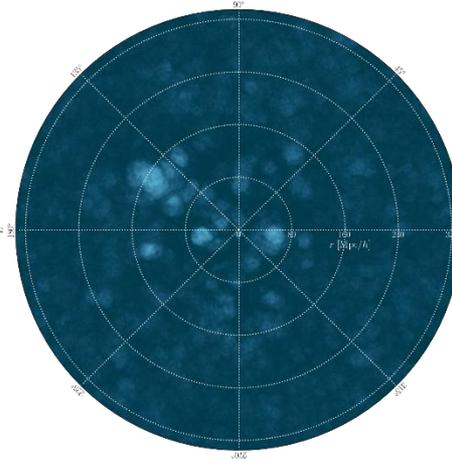
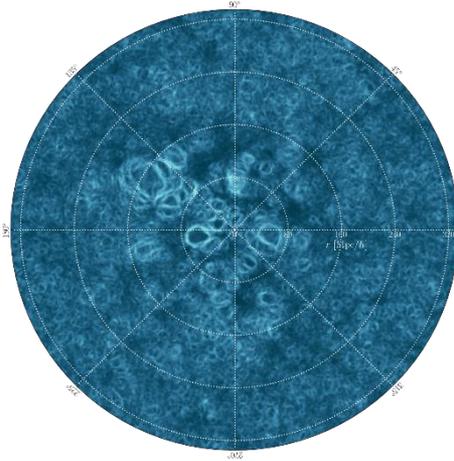
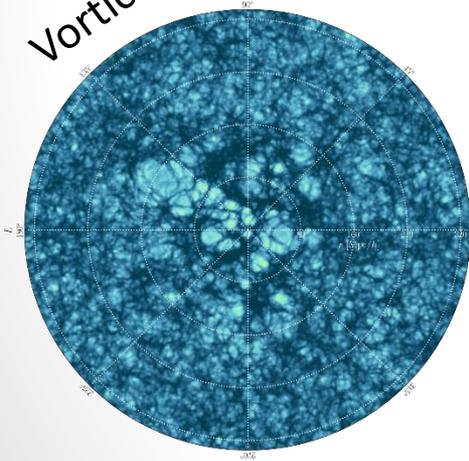
Sheets



Voids



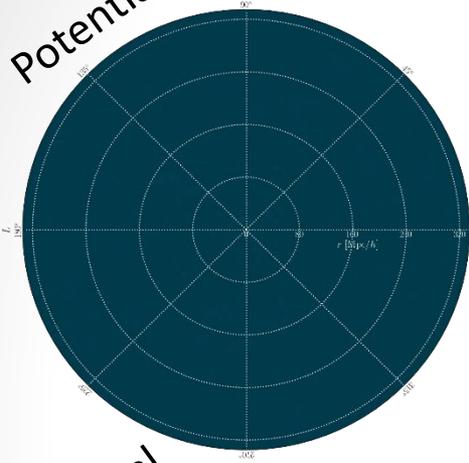
Vortical



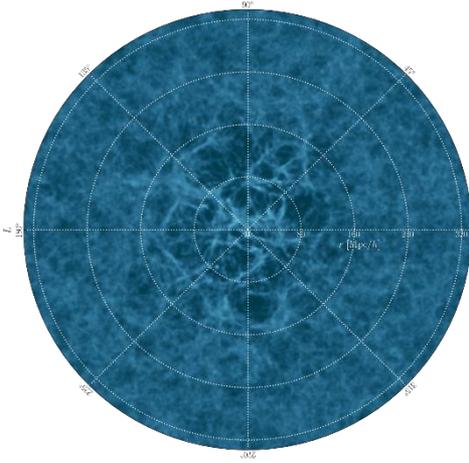
FL, Lavaux & Jasche, in prep.

LICH final structures inferred by BORG

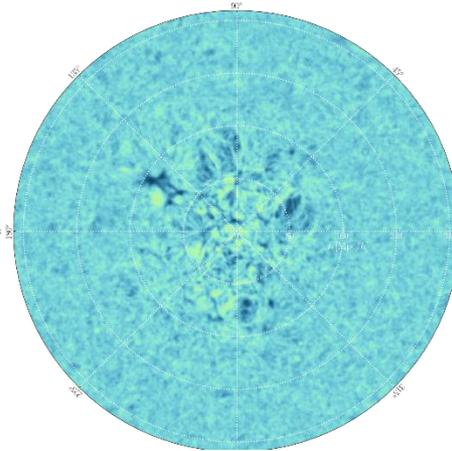
Potential Clusters



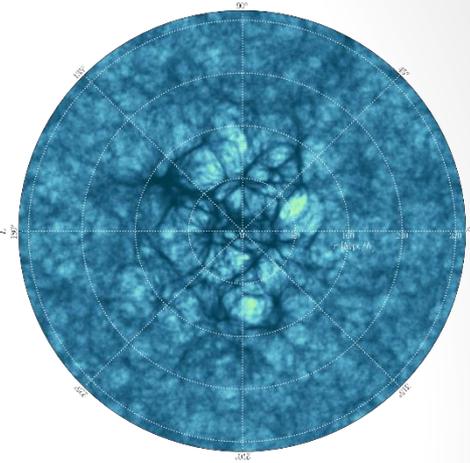
Filaments



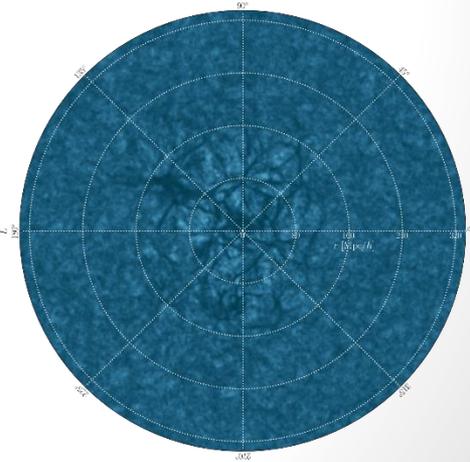
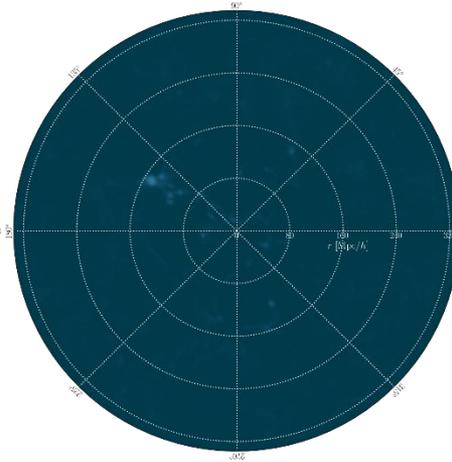
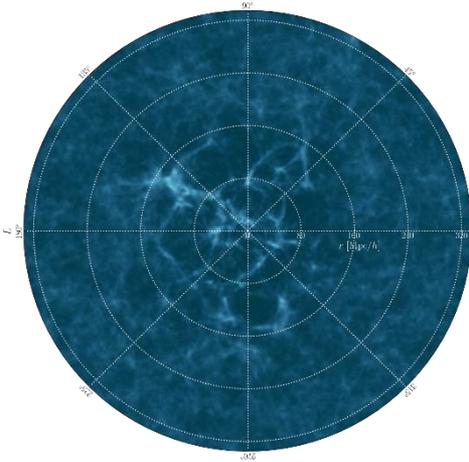
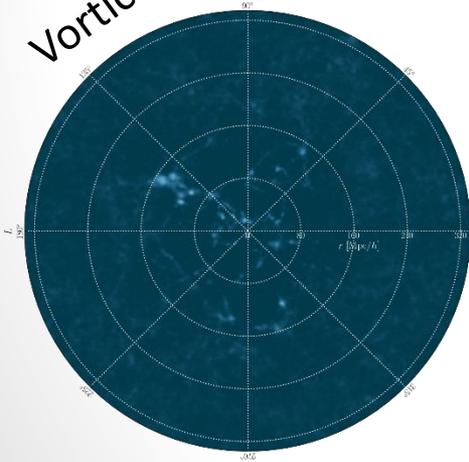
Sheets



Voids



Vortical



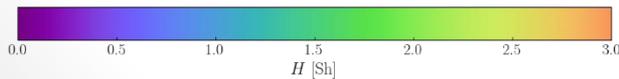
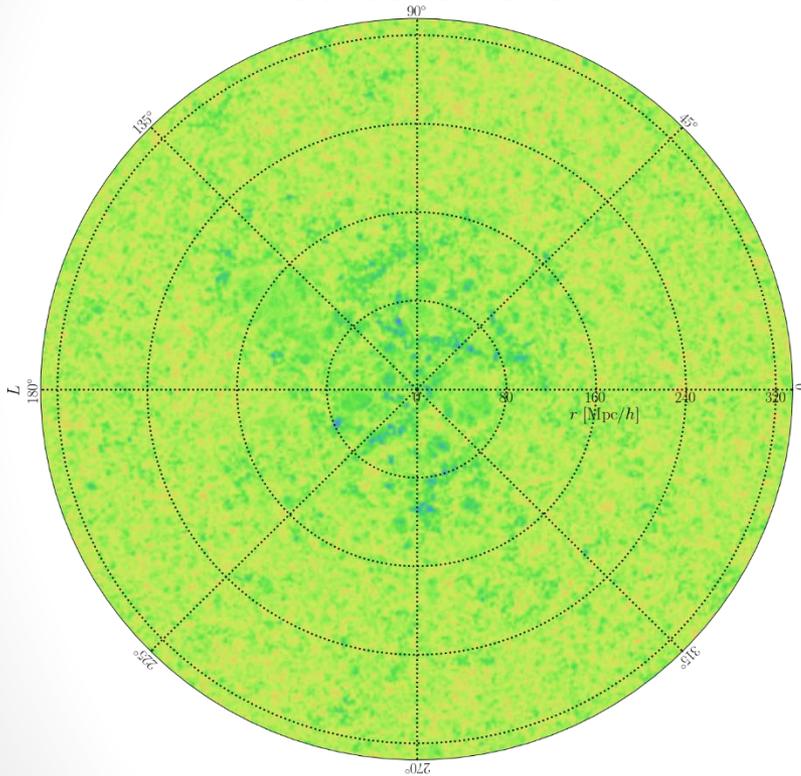
FL, Lavaux & Jasche, in prep.

How is information propagated?

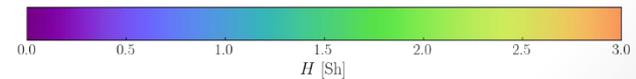
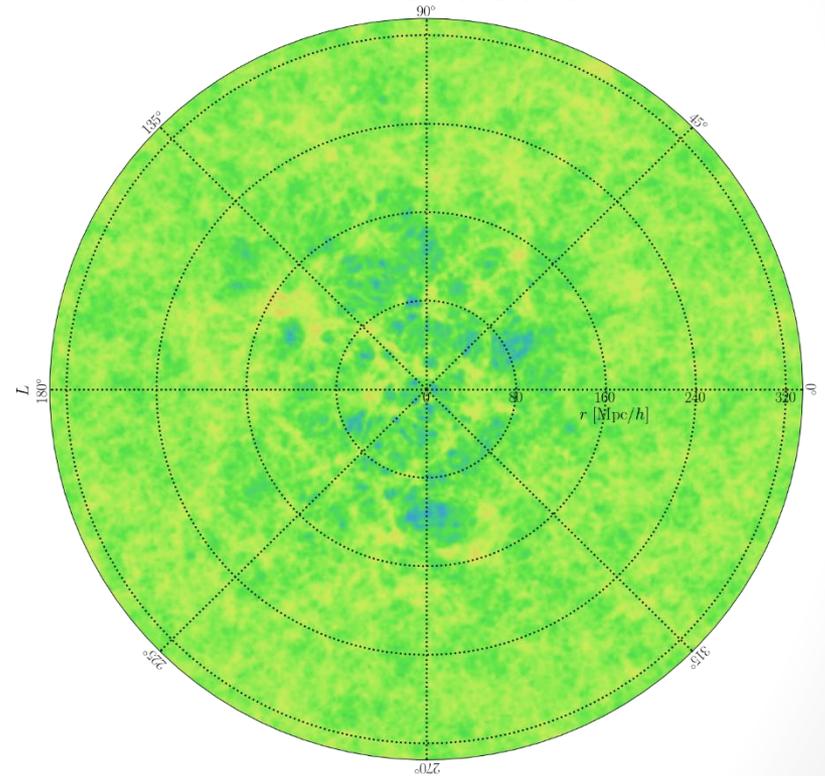
Shannon entropy:

$$H[\mathcal{P}(T(\vec{x})|d)] \equiv - \sum_{i=0}^7 \mathcal{P}(T_i(\vec{x})|d) \log_2(\mathcal{P}(T_i(\vec{x})|d)) \quad \text{in shannons (Sh)}$$

Initial conditions



Final conditions



FL, Lavaux & Jasche, in prep.

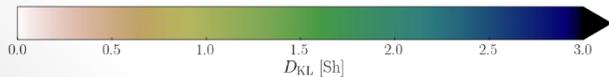
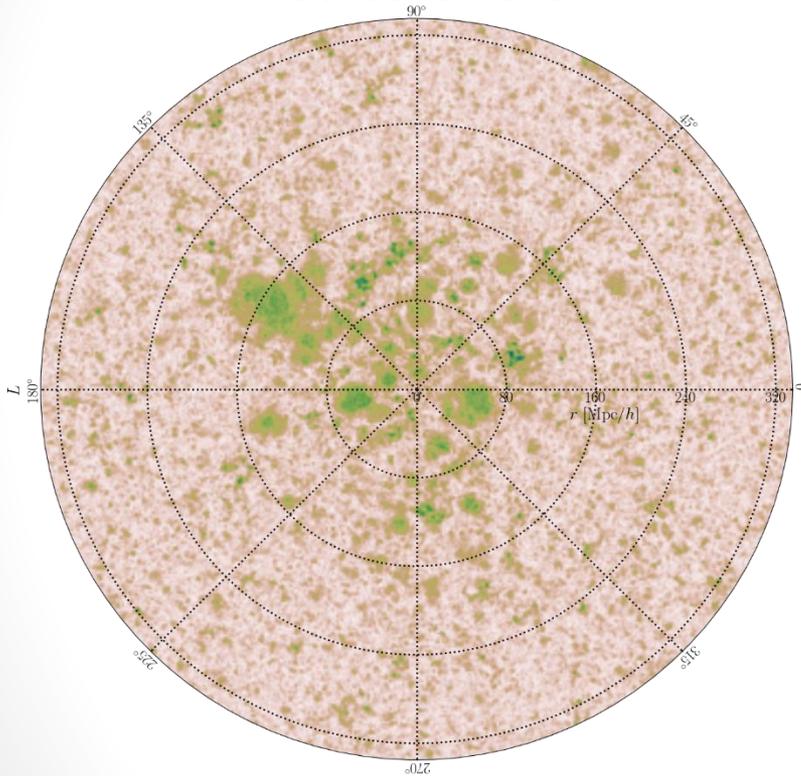
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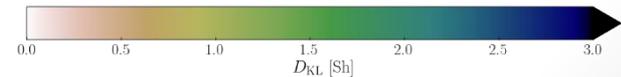
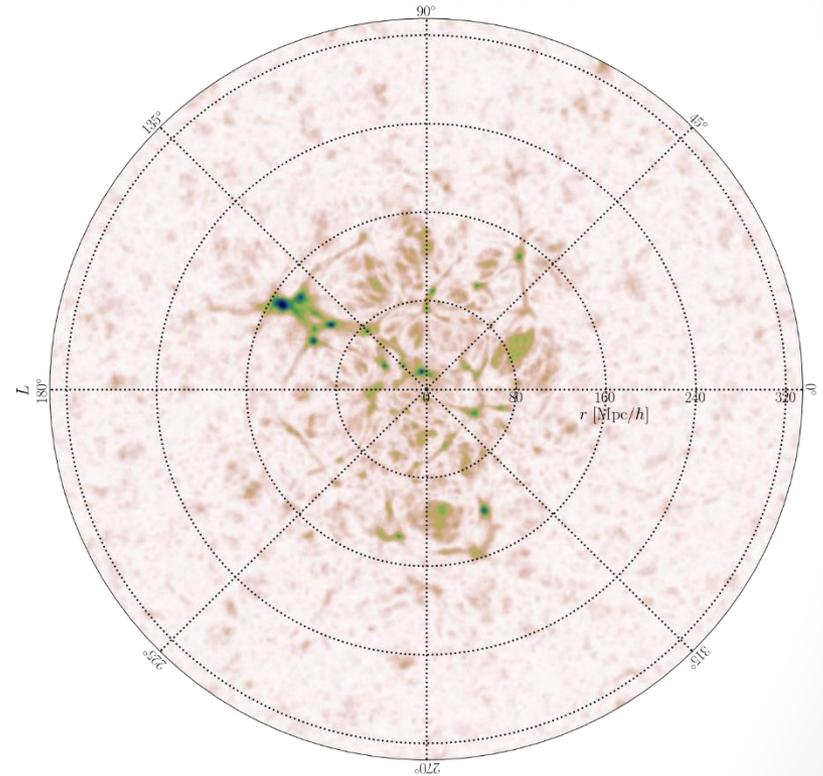
How much did the data surprise us? Information gain:

$$D_{\text{KL}}[\mathcal{P}(\mathbf{T}(\vec{x})|d)||\mathcal{P}(\mathbf{T})] \equiv - \sum_{i=0}^7 \mathcal{P}(\mathbf{T}_i(\vec{x})|d) \log_2 \left(\frac{\mathcal{P}(\mathbf{T}_i(\vec{x})|d)}{\mathcal{P}(\mathbf{T}_i)} \right) \text{ in shannons (Sh)}$$

Initial conditions



Final conditions



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A decision rule for structure classification

- Space of 8 “input features”:

$\{T_0 = \text{potential void}, T_1 = \text{potential sheet}, T_2 = \text{potential filament},$
 $T_3 = \text{potential cluster}, T_4 = \text{vortical void}, T_5 = \text{vortical sheet},$
 $T_6 = \text{vortical filament}, T_7 = \text{vortical cluster}\}$

- Space of 9 “actions”:

$\{a_j = \text{“decide structure } T_j\text{” for } 0 \leq j \leq 7,$
 $a_{-1} = \text{“remain undecided”}\}$

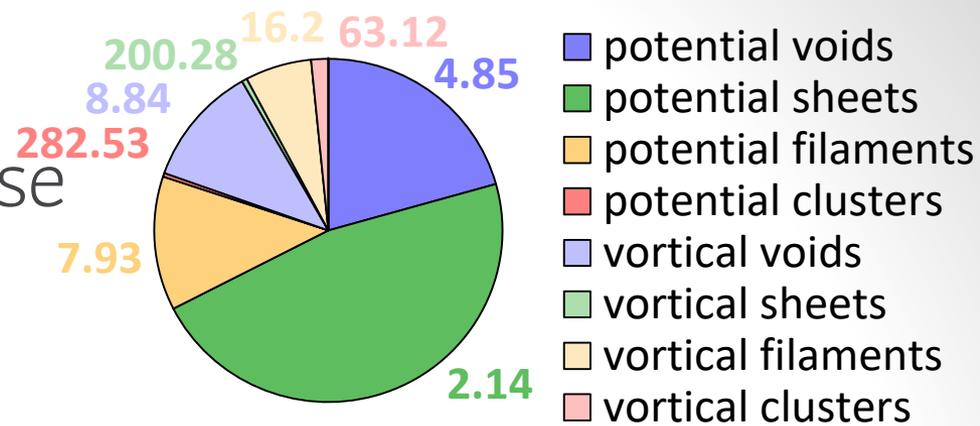
➡ A problem of **Bayesian decision theory**:

one should take the action that maximises the utility

$$U(a_j(\vec{x})|d) = \sum_{i=0}^7 G(a_j|T_i) \mathcal{P}(T_i(\vec{x})|d)$$

- How to write down the gain functions?

Gambling with the Universe



- One proposal:

$$G(a_j | T_i) = \begin{cases} \frac{1}{\mathcal{P}(T_i)} - \alpha & \text{if } j \in \llbracket 0, 7 \rrbracket \text{ and } i = j & \text{“Winning”} \\ -\alpha & \text{if } j \in \llbracket 0, 7 \rrbracket \text{ and } i \neq j & \text{“Losing”} \\ 0 & \text{if } j = -1. & \text{“Not playing”} \end{cases}$$

- Without data, the expected utility is

$$U(a_j) = 1 - \alpha \quad \text{if } j \neq -1 \quad \text{“Playing the game”}$$

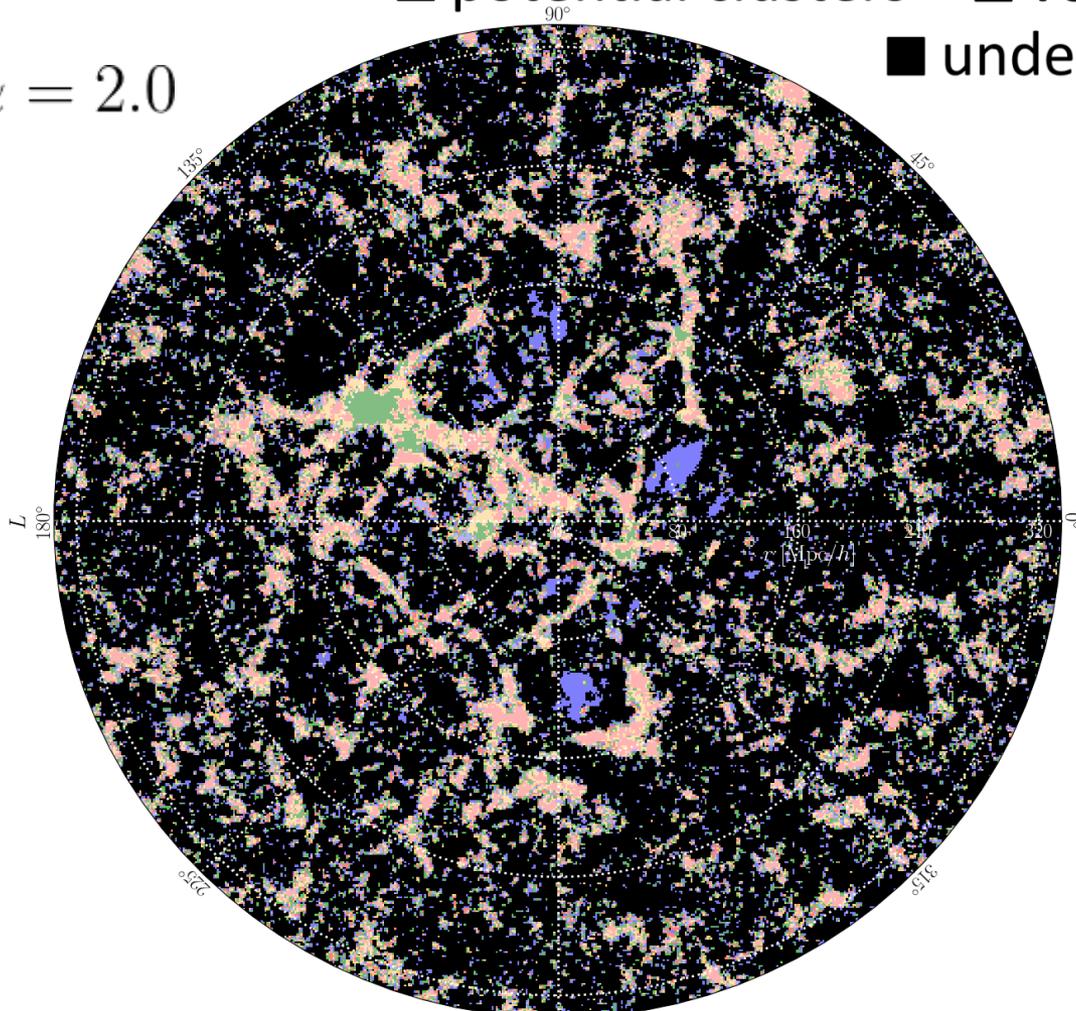
$$U(a_{-1}) = 0 \quad \text{“Not playing the game”}$$

- With $\alpha = 1$, it's a *fair game* → always play
→ “speculative map” of the LSS
- Values $\alpha > 1$ represent an *aversion for risk*
→ increasingly “conservative maps” of the LSS

Playing the game...

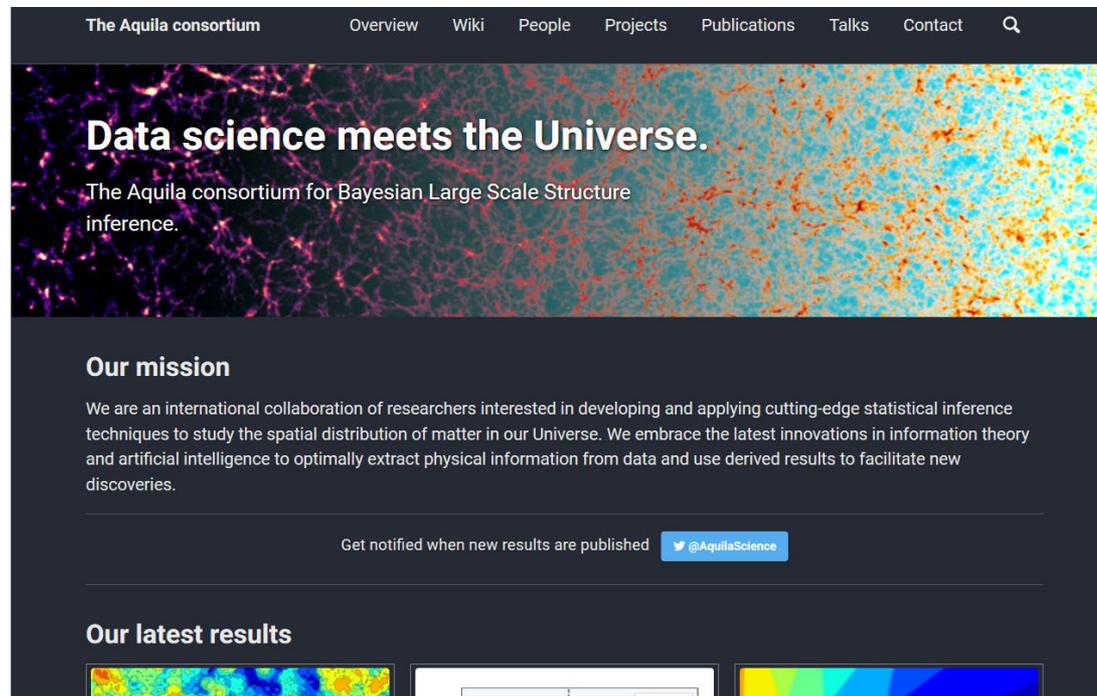
- potential voids
- potential sheets
- potential filaments
- potential clusters
- vortical voids
- vortical sheets
- vortical filaments
- vortical clusters
- undecided

$\alpha = 2.0$



The Aquila Consortium

- Created in 2016. Members from the UK, France, Germany & Sweden.
- Gathers people interested in developing the Bayesian pipelines and running analyses on cosmological data.



www.aquila-consortium.org

Conclusions

- **BORG** is a **Bayesian inference engine** allowing the analysis of the **large-scale structure** and its formation history.
- Thanks to **BORGPM**, the **cosmic web** can be described using **LICH**, a classifier distinguishing potential and vortical flows.
- A probabilistic analysis of the cosmic web yields a data-supported **connection between cosmology and information theory**.
- **Decision theory** offers a framework to **classify structures** in the presence of data constraints and uncertainty.